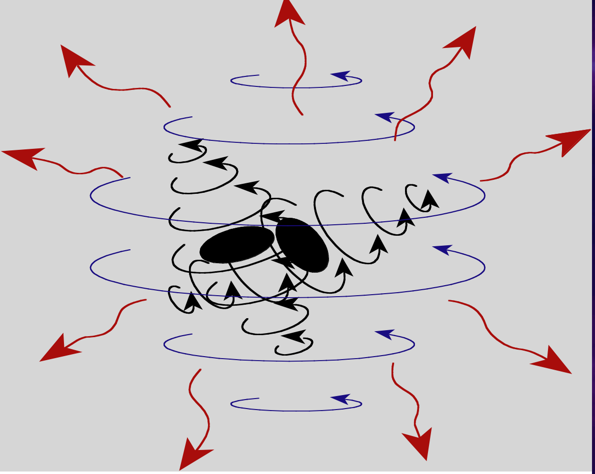
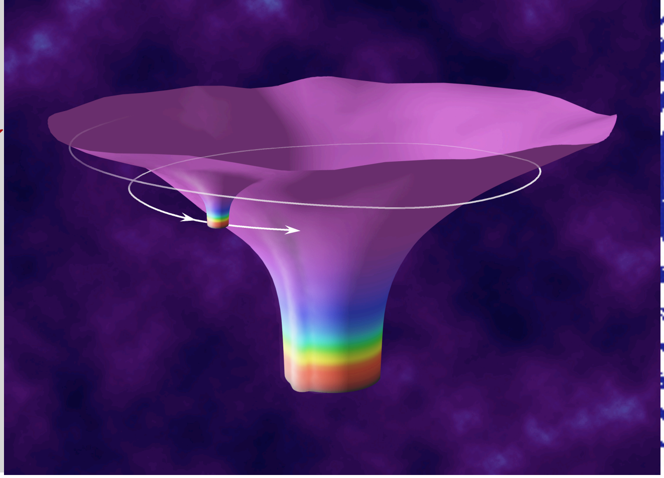
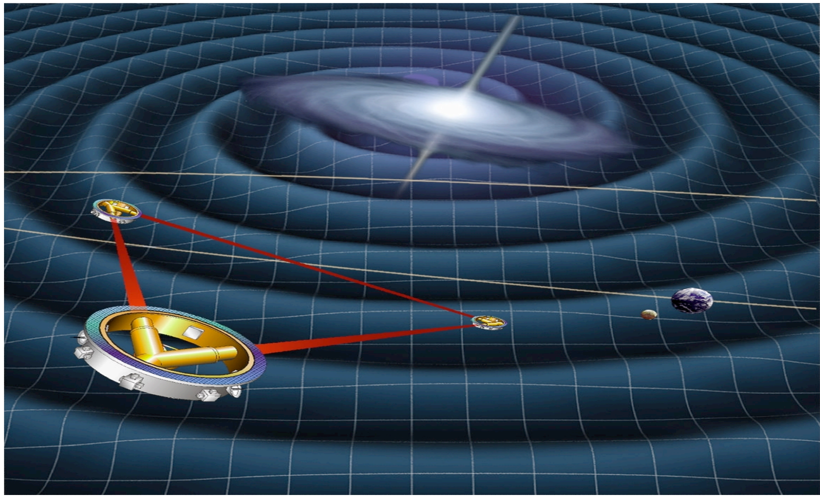
  

Black Holes

Kip Thorne

Chandrasekhar Centenial Symposium

University of Chicago, 15 September 2010

What I Will NOT Cover

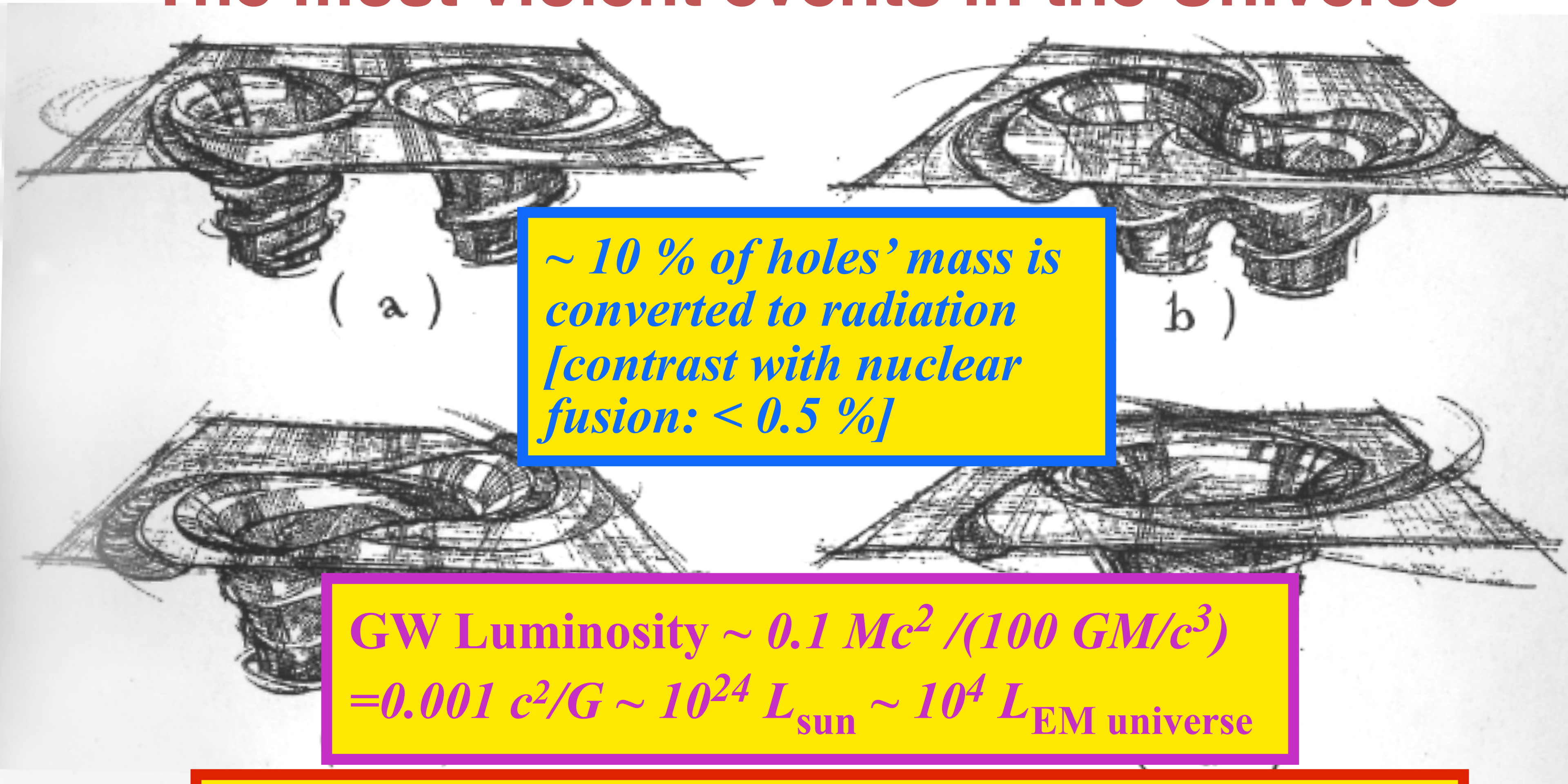
- Semi-Classical Aspects of Black Holes
- Quantum Aspects of Black Holes
- Higher-Dimensional Black Holes
- Astrophysics of Black Holes

  [talk by Priyamvada Natarajan tomorrow]

What I Will Cover

- Black Holes in the context of
  - » Numerical Relativity
  - » Gravitational Wave Observations
- A new “golden age” of black-hole research

The “Holy Grail”: Collisions of Black Holes - The most violent events in the Universe

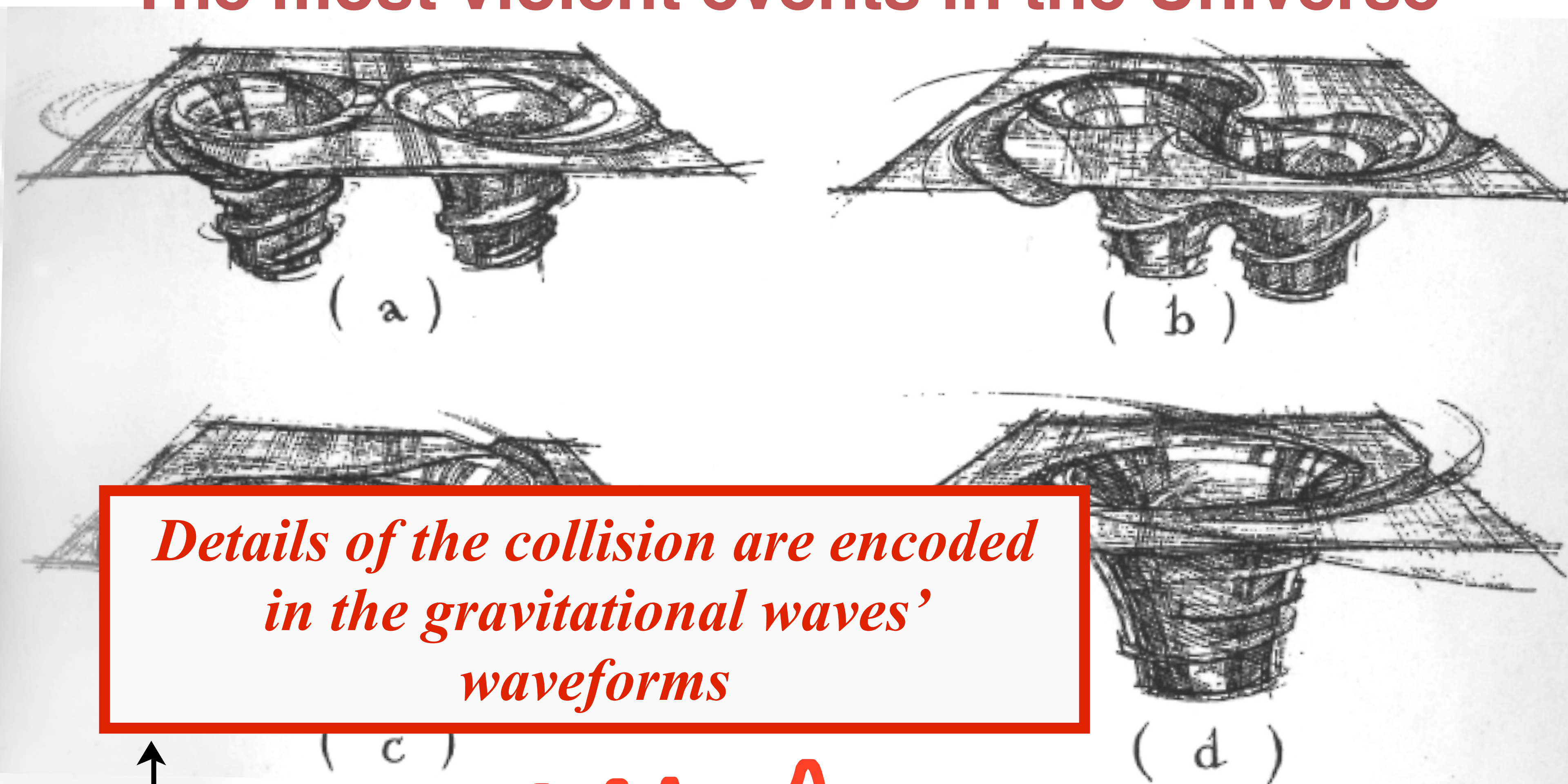


~ 10 % of holes’ mass is converted to radiation [contrast with nuclear fusion: < 0.5 %]

GW Luminosity ~ $0.1\, Mc^2 /(100\, GM/c^3)$ $=0.001\, c^2/G \sim 10^{24}\, L_{\mathrm{sun}} \sim 10^4\, L_{\mathrm{EM\ universe}}$

No Electromagnetic Waves emitted whatsoever - except from, e.g. disturbed accretion discs

Collisions of Black Holes: The most violent events in the Universe



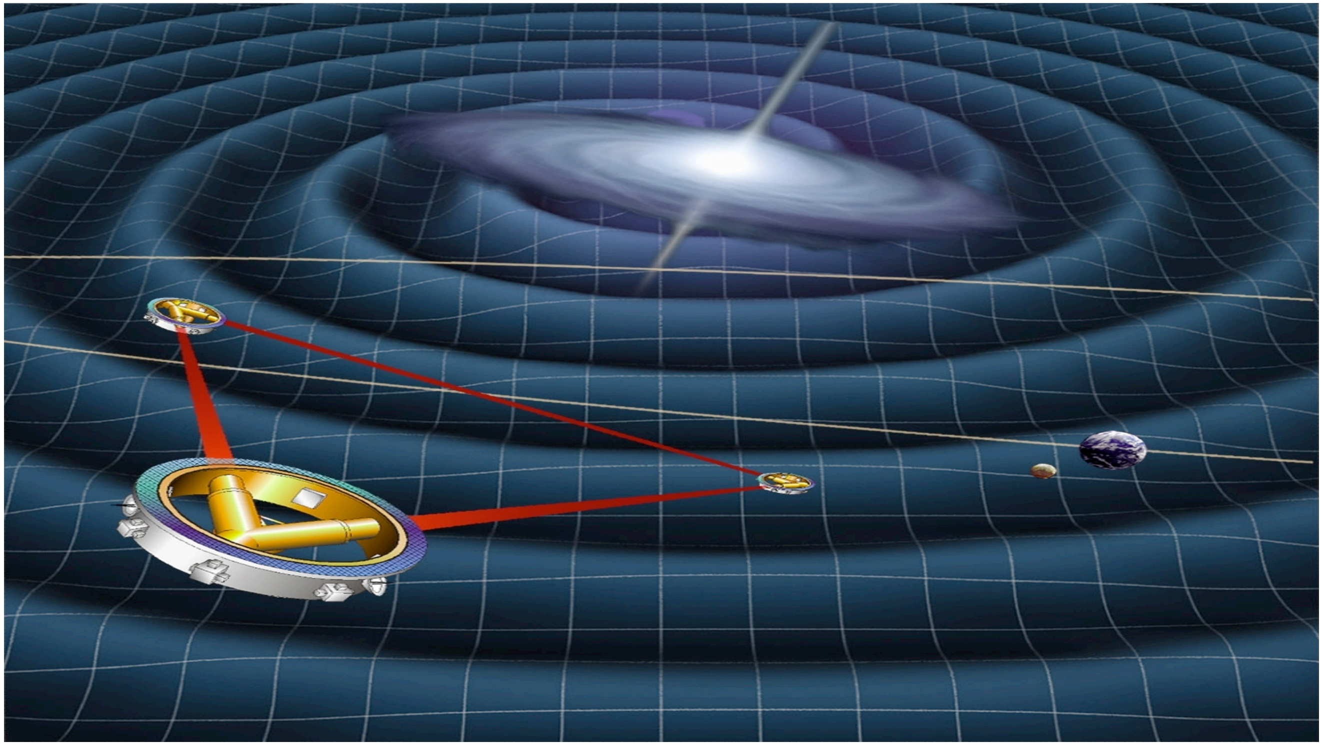
Details of the collision are encoded in the gravitational waves' waveforms

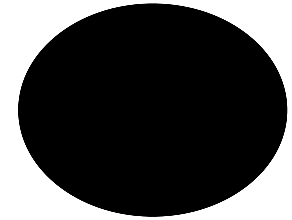
The Context: *LIGO*

Black Hole Masses:
~2 to 1000 Msun

- ~700 scientists, ~ 60 Institutions, 12 nations,
- Initial detectors and searches: 2005-2010
- Advanced detectors: Begin installation next Wednesday
- All data analyzed jointly with VIRGO

The Context: *LISA*



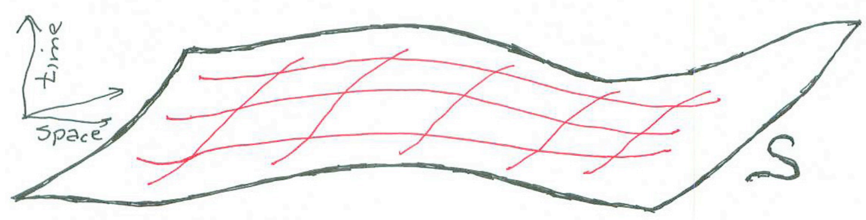


Black Hole Masses: ~10,000 Msun to ~10 million Msun

- Joint NASA / ESA Mission
- LISA Pathfinder (technology test) mission: 2013
- LISA launch: ~2022?

Numerical Relativity: How is it Done?

- Evolve the geometry of spacetime - not fields in spacetime
- Choose an initial spacelike 3-dimensional surface *S*
  - » Put a coordinates on *S*



- Specify: *3-metric* g_{ij} and *Extrinsic Curvature* K_{ij} of S
  - » Subject to *constraint equations* [analogues of Div B = 0]
- Lay out coordinates to future by specifying Lapse function α and Shift function β^i
- Integrate 3-metric forward in time via *dynamical equations*

$$ds^2 = -\alpha^2 dt^2 + g_{ij} (dx^i - \beta^i dt) (dx^j - \beta^j dt)$$

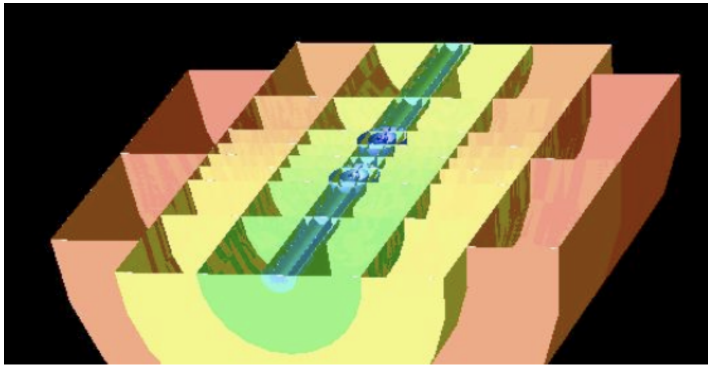
Numerical Relativity

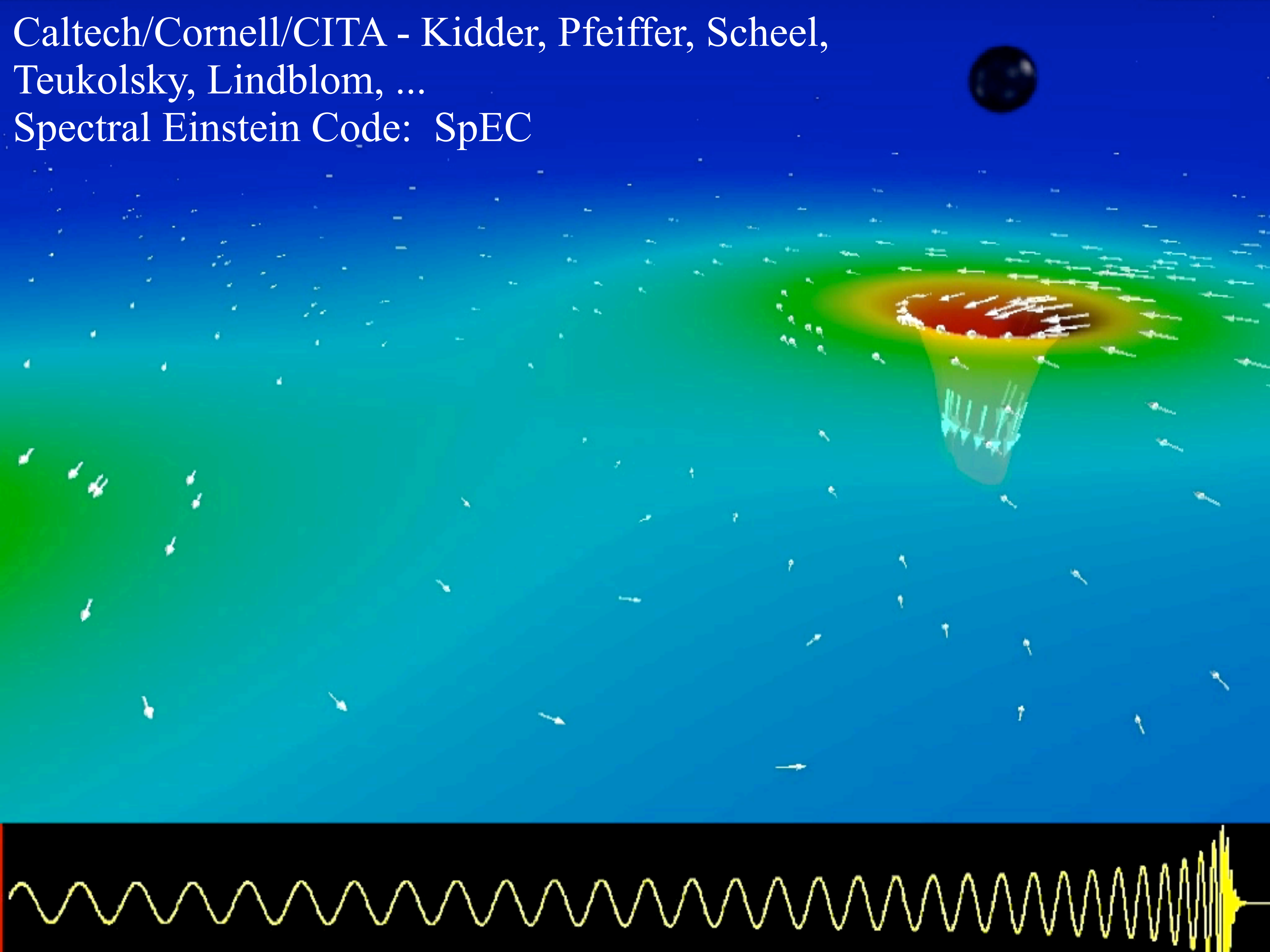
Two Major Pitfalls

- Constraint-Violation Instabilities:
  - » Slight initial error in constraints (analog of Div B = 0) blows up in time
  - » Solved after ~ 5 years of struggle
- Coordinates become singular [“gauge instability”]
  - » Only recently solved *robustly* in highest-precision codes

Two Mature Approaches

- Finite-difference
  - » Robust, power-law convergence
- Spectral
  - » More complicated, less robust
    - but exponential convergence




Caltech/Cornell/CITA - Kidder, Pfeiffer, Scheel,
Teukolsky, Lindblom, ...
Spectral Einstein Code: SpEC

Current NR BBH Capabilities

- Many groups can simulate generic black holes (unequal masses, spins with random orientations)
  - Princeton (Pretorius), Rochester Institute of Technology (Campanelli, ...), Goddard Spaceflight Center (Centrella, ...), U. Illinois (Shapiro, ...), Albert Einstein Instititute & LSU (Pollney, ...), U. Jena (Bruegmann, ...), Georgia Tech (Laguna, ...), U. Texas (Matzner, ...), U. Maryland (Tiglio, ...), Florida Atlantic U. Tichy, ...), Barcelona (Sperhake, ...), Cornell/Caltech/CITA (Teukolsky, ...), ...

Current NR BBH Capabilities

- Mass ratios:
  - » most groups: up to 6:1
  - » Highest [RIT - Campanelli, Lousto, Zlochower]: 100:1
- Spin magnitudes, $\chi = S/M^2$
  - » most groups: up to 0.6
  - » Highest for binary: 0.95 [Cornell/Caltech/CITA, Lovelace, Scheel, Szilagyi]
    - – standard method for solving constraint equations [Bowen/York] limited to $\chi < 0.93$
    - – new method: superposed Kerr-Schild

"S-Matrix" Insights from NR

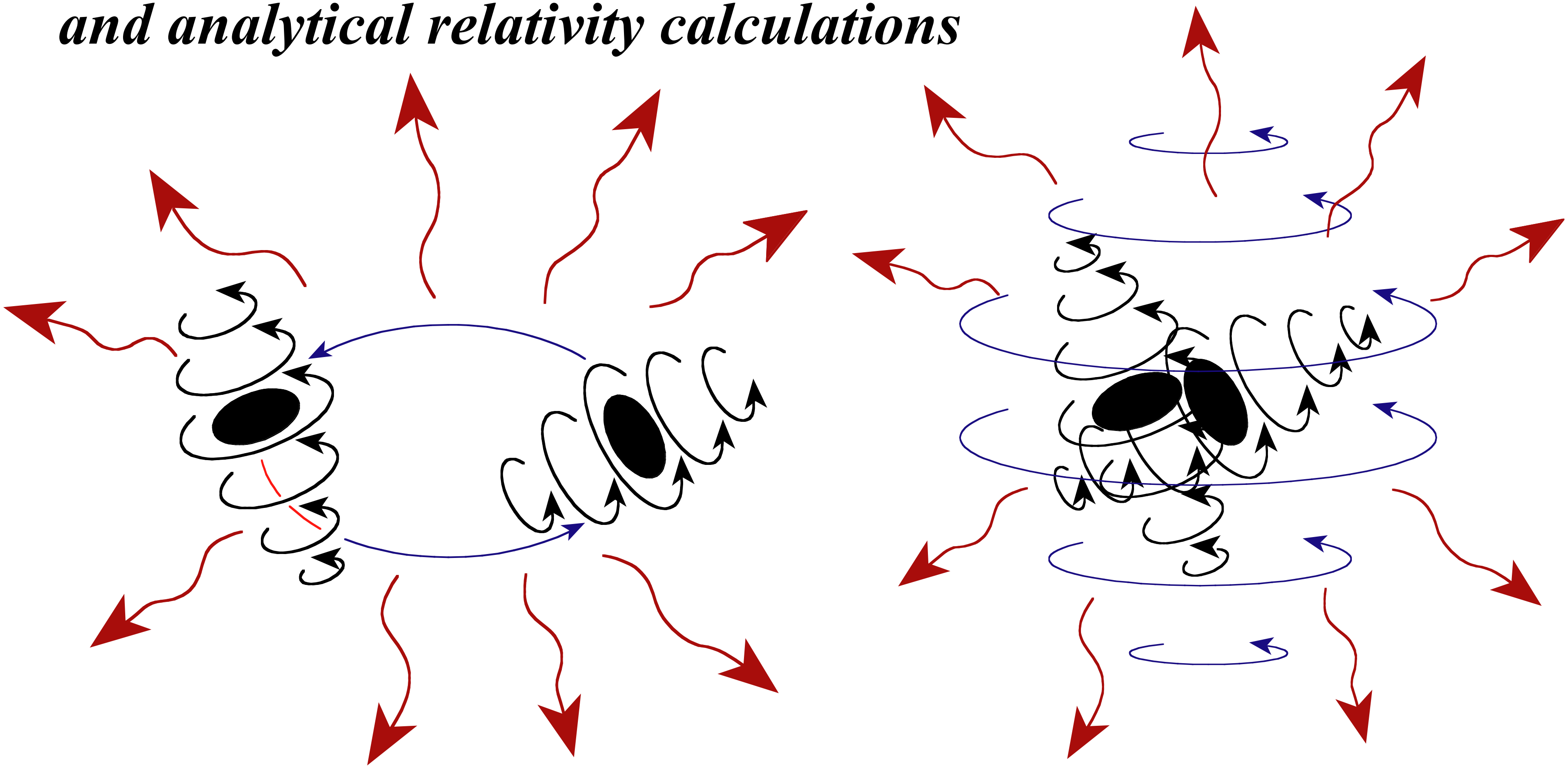
- S-Matrix:
  - » Initial state: Masses, Spins, Orbit
  - » Final state: Mass, Spin, Kick Velocity, Gravitational Waveforms

BBH Waveforms for LIGO
Clifford Will's Talk

- Waveforms needed in analytic form [quickly computable, "on the fly"] for searches via "matched filter method"
  - » Accuracy needed [Lindblom, Owen, brown]: $\delta\phi \sim \delta A/A \sim 0.1$ for searches, ~ 0.01 for information extraction
  - » PN at early times; NR at late times; match
  - » NR simulations
    - – 7 parameters: M_2/M_1, $\boldsymbol{S}_2$, $\boldsymbol{S}_1$
    - – need ~1000 simulations
  - » Use PN and NR to "tune" parametrized analytical formulae, e.g. "Effective One Body"
- Underway: Large collaboration led by Buonanno [U Md] - most of the world's NR groups + several Analytical Relativity groups

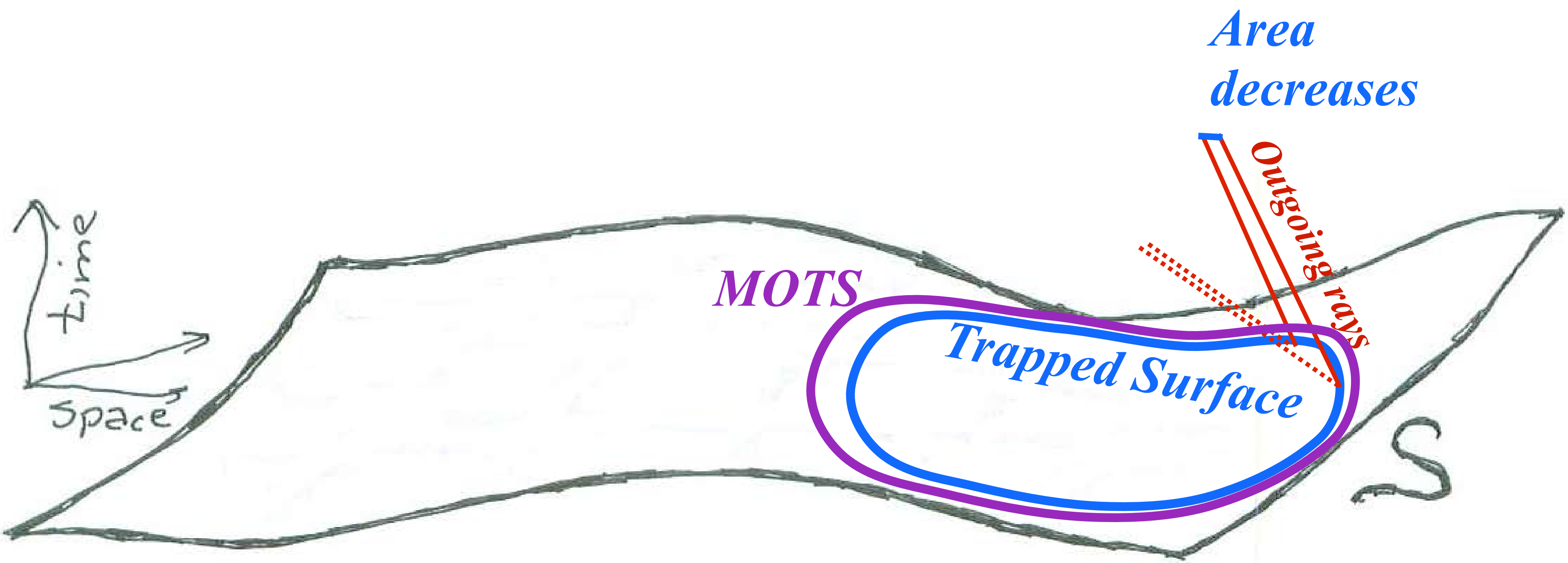
Nonlinear Dynamics of Curved Spacetime

What we are learning from combined NR simulations and analytical relativity calculations



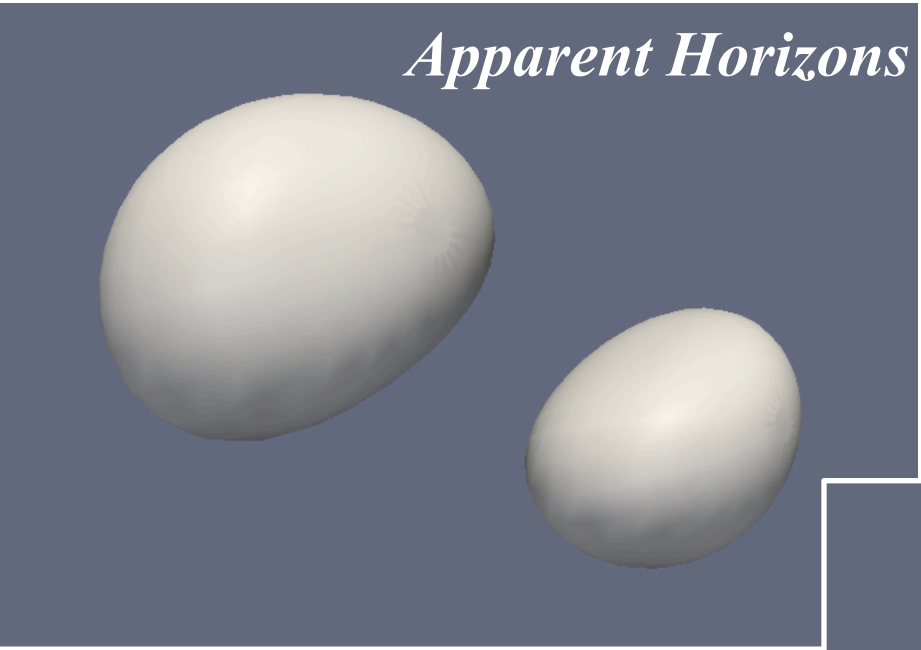
Horizons **Vortexes** **Tendexes**

Marginally Outer Trapped Surface (MOTS) and Apparent Horizon [Penrose]



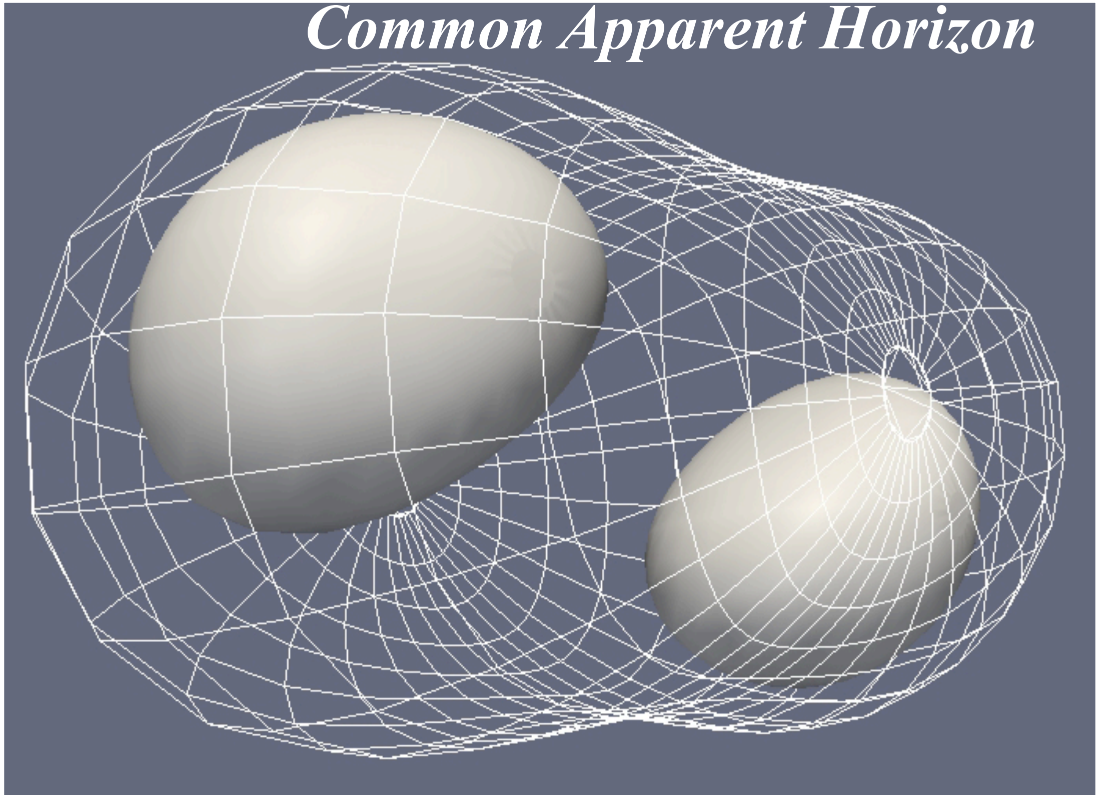
Outermost MOTS: Apparent Horizon

From SpEC Simulation



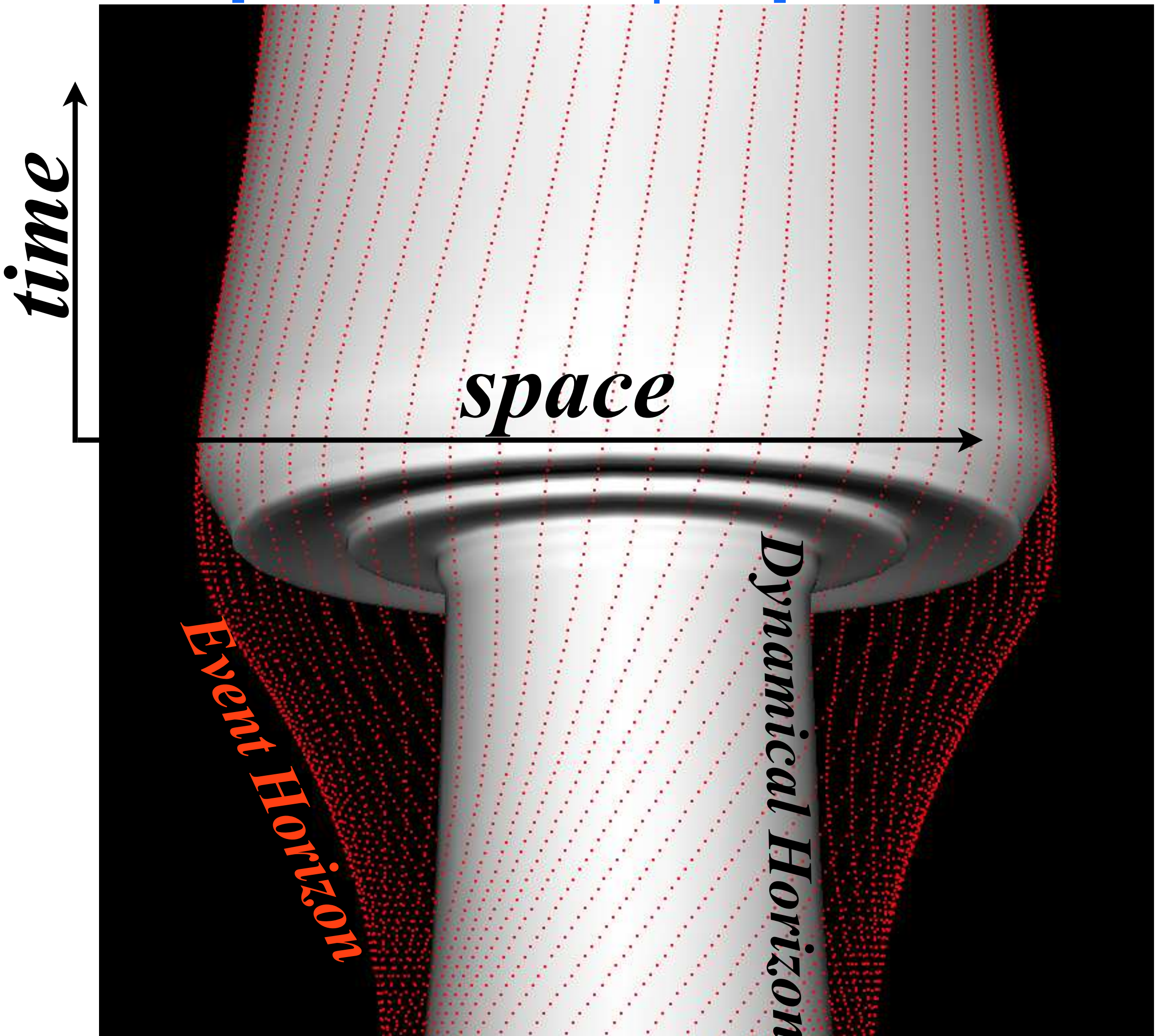
Rob Owen [Cornell]

Nested MOTSs



Apparent Horizon vs Event Horizon

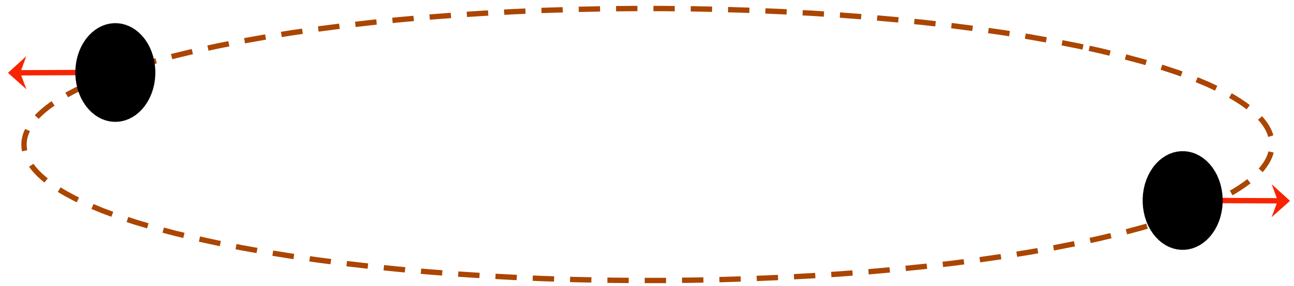
- Tony Chu, Harald Pfeiffer, Michael Cohen:
  - Kerr black hole perturbed by a very strong pulse of ingoing gravitational waves [evolved with SpEC]



- *Dynamical horizon [Ashtekar, Krishnan]:* spacelike world tube of *marginally outer trapped surfaces (MOTS)*
- *Apparent horizon [Penrose]:* outermost MOTS
- *Event Horizon [Hawking]:* boundary of communication

Spinning Black Holes

Rochester Institute of Technoogy:
Campanelli, Lousto, Zlochower




Simulation:
Manuela Campanlli
Carlos Lousto
Yosef Zlochower
Visualization:
Hans-Peter Bischof
CCRG
RIT
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Spinning Black Holes

Rochester Institute of Technoogy:
Campanelli, Lousto, Zlochower

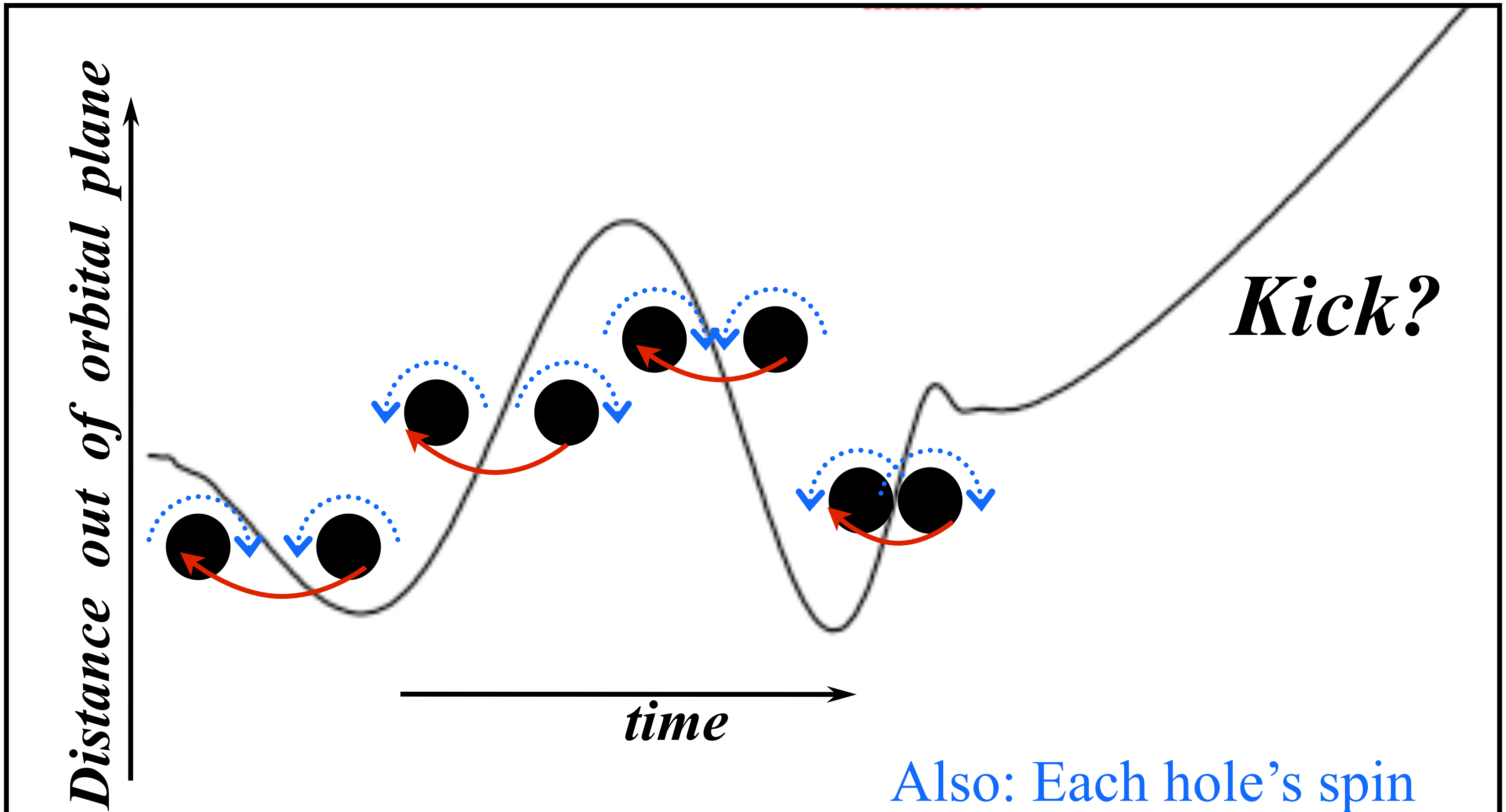
Weak gravity:

$$ds^2 = -(1 - 2M/r)dt^2 + dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta (d\phi - \omega dt)^2$$

$$\omega = 2J/r^3$$
$$\boldsymbol{v}_{\mathrm{FD}} = \omega r \boldsymbol{e}_{\hat{\phi}}$$

Analogous to 2 Vortices in a Fluid

Explanation of Bobbing
[Pretorius]

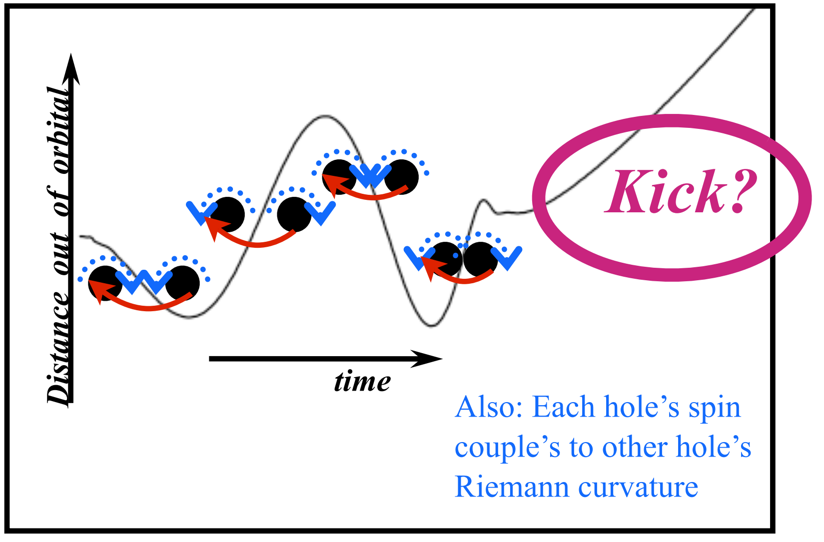


Also: Each hole's spin couple's to other hole's Riemann curvature

Other new tools:
Momentum flow - Chen, Keppel, ...;
Gralla, Harte, Wald

New Insights Into Nonlinear Dynamics of Curved Spacetime

- ***Cornell/Caltech/CITA Numerical Relativity Group***
  - *Teukolsky, Kidder, Lindblom, Pfeiffer, Szilagyi, Scheel, Owen, Lovelace, Kaplan, Matthews, ...*

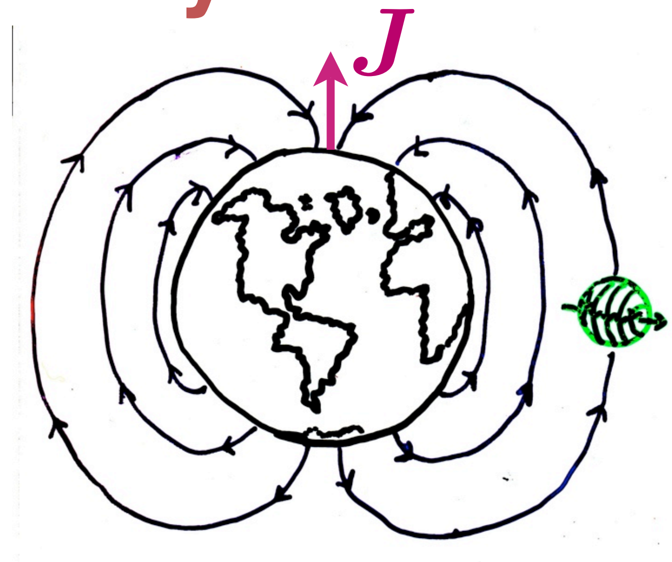


- ***Caltech Analytical Relativity Theory (CaRT) Group***
  - *Chen, Nichols, Thorne, ...*

Frame Dragging When Gravity is Weak

- Gyroscope precession relative to distant stars [inertial frames at "infinity"]

$$\boldsymbol{\Omega}_{\rm fd} = \frac{1}{2}\boldsymbol{\nabla} \times \boldsymbol{v}_{\rm fd} = -\left[\frac{\boldsymbol{J} - 3(\boldsymbol{J}\cdot\boldsymbol{n})\boldsymbol{n}}{r^3}\right]$$

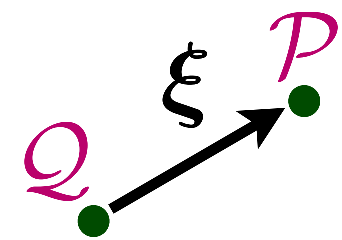


- When gravity is strong and dynamical: cannot use inertial frames at infinity.

- Differential frame dragging: Precession angular velocity at $\mathcal{P}$ relative to inertial frames at $\mathcal{Q}$

$$\Delta\boldsymbol{\Omega}_{\rm fd} = \boldsymbol{\xi}\cdot\boldsymbol{\nabla}\boldsymbol{\Omega}_{\rm fd}$$

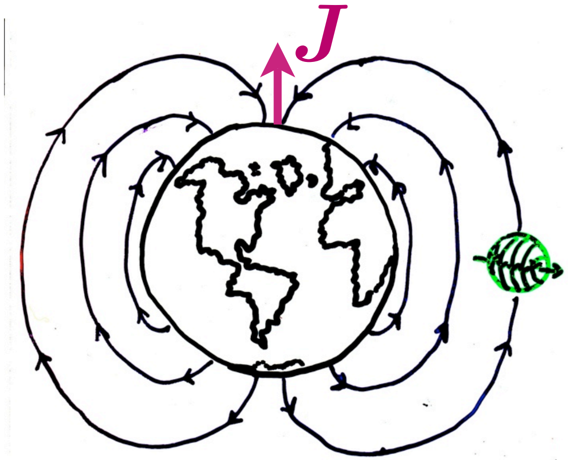
$\boldsymbol{\mathcal{B}}$ "Magnetic" part of Weyl Curvature Tensor

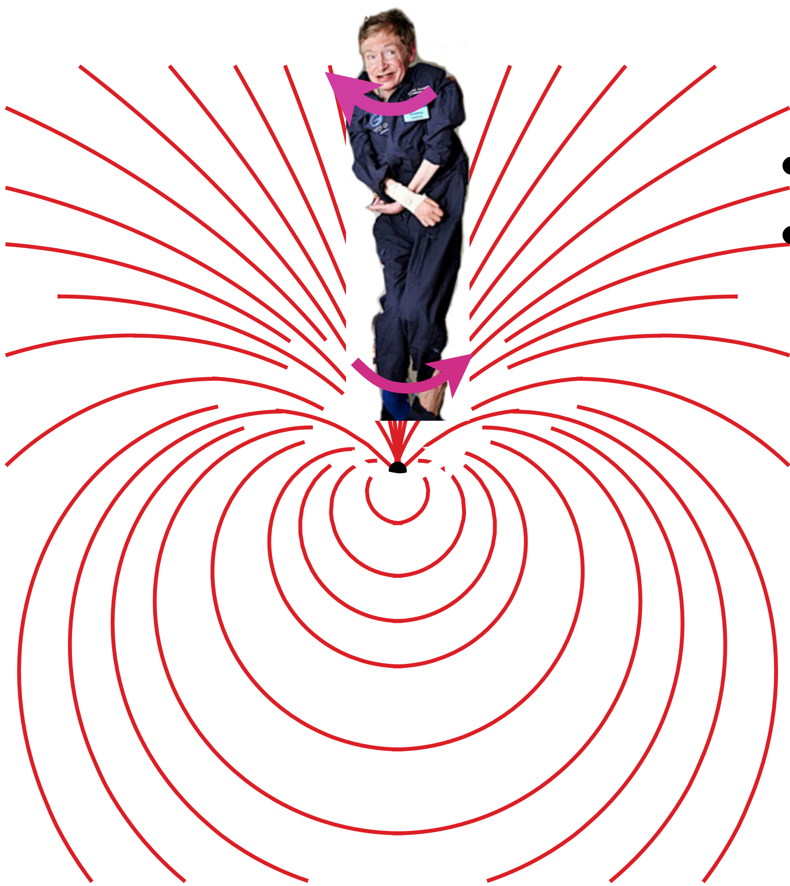


- $\mathcal{B}_{jk}$ symmetric and trace free. Determined by 3 orthogonal eigendirections (principal axes) and their eigenvalues.

Frame Dragging When Gravity is Weak

- *Frame-Dragging* ***Vortex Line***: Streamline of eigenvector $\boldsymbol{n}$ of $\mathcal{B}_{jk}$
- ***Vorticity*** *of Line:* Eigenvalue $\mathcal{B}_{nn}$



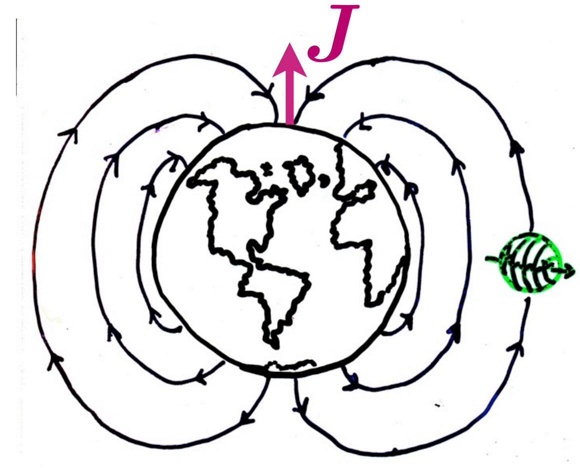


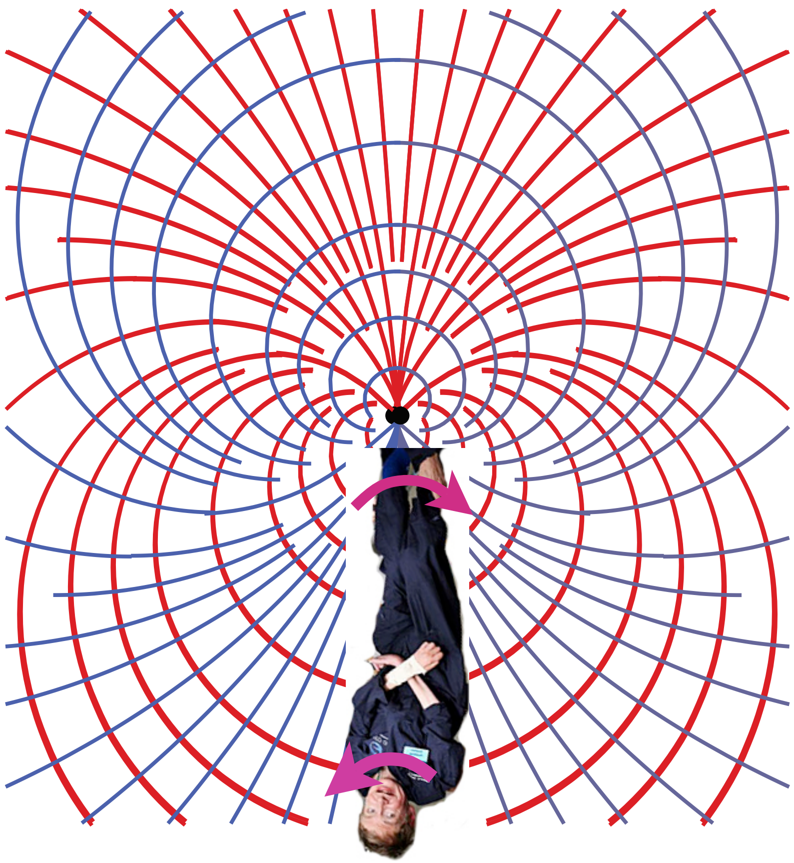
- Head sees feet dragged counter-clockwise
- Feet see head dragged counter-clockwise

negative-vorticity vortex lines

Frame Dragging When Gravity is Weak

- *Frame-Dragging **Vortex Line**:* Streamline of eigenvector $\boldsymbol{n}$ of $\mathcal{B}_{jk}$
- ***Vorticity** of Line:* Eigenvalue $\mathcal{B}_{nn}$



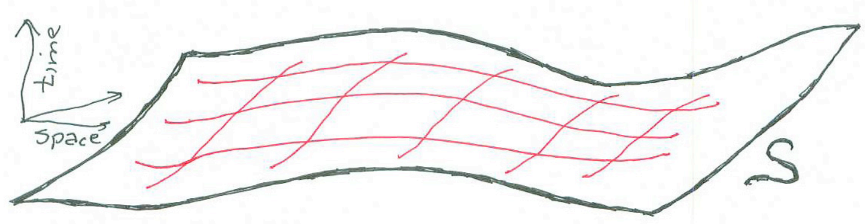


- Head sees feet dragged clockwise
- Feet see head dragged clockwise

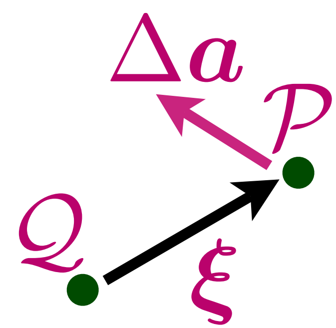
positive-helicity
vortex lines

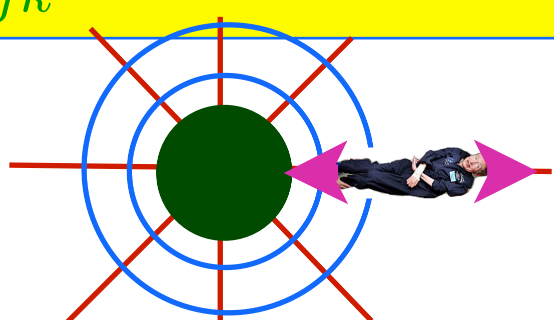
Frame Dragging and Tidal Pull When Gravity is Strong

- Slice spacetime into space plus time



- Observers who move orthogonal to space slices S decompose the spacetime curvature (vacuum Riemann tensor) into "electric" part $\mathcal{E}_{jk}$ and "magnetic" part $\mathcal{B}_{jk}$
- $\mathcal{B}_{jk}$ describes differential frame dragging - with *vortex lines* and their *vorticities*
- $\mathcal{E}_{jk}$ describes tidal accelerations $\Delta a_j = -\mathcal{E}_{jk}\,\xi^k$
- $\mathcal{E}_{jk}$: *tendex* lines and their *tendicity*

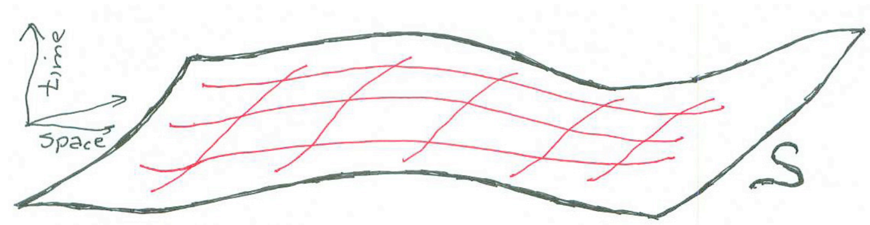




$\mathcal{E}_{nn} < 0$ *tidal stretch*

Frame Dragging and Tidal Pull When Gravity is Strong

- Slice spacetime into space plus time



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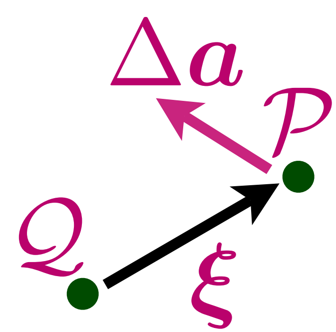
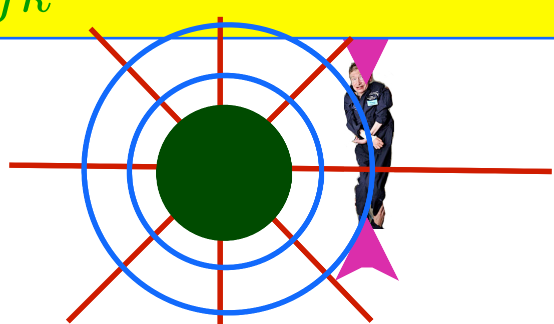
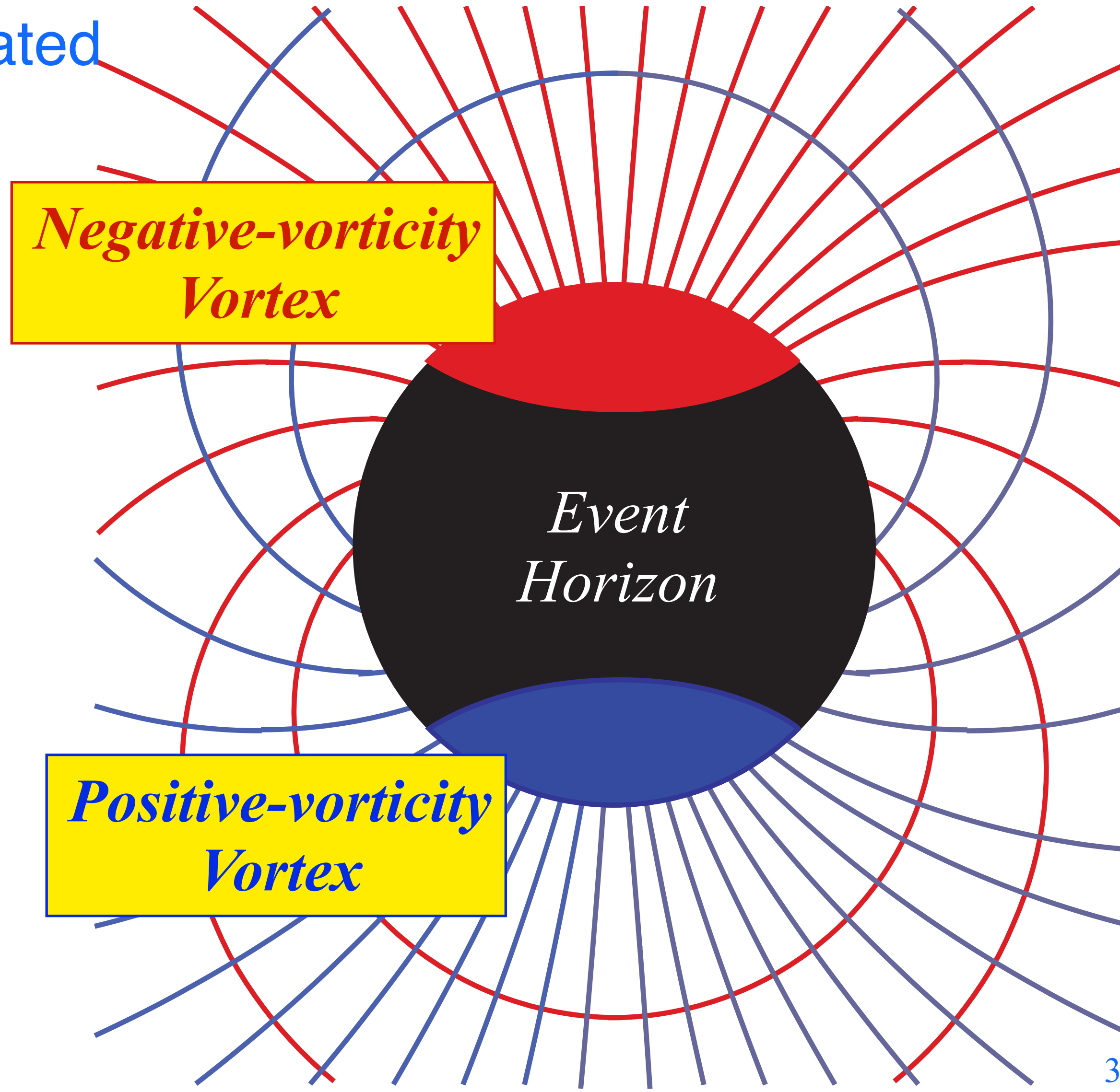




$\mathcal{E}_{nn} > 0$ *tidal squeeze*

Quiescent, Spinning (Kerr) Black Hole

- *Frame-Dragging **Vortex**:* Region of concentrated vorticity

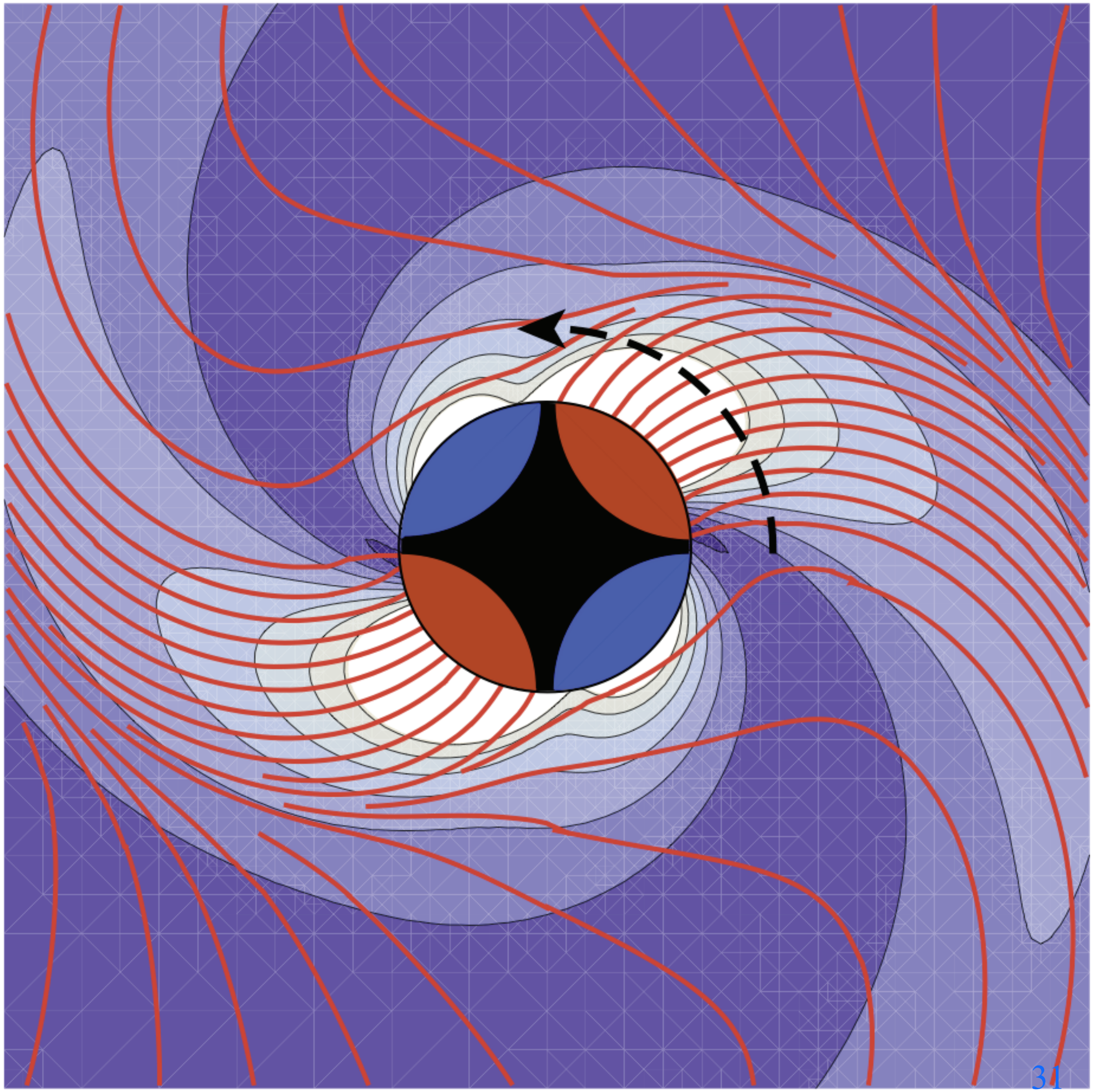


Normal-mode Vortexes Traveling Around a Nonrotating hole

l=m=2, odd-parity normal mode

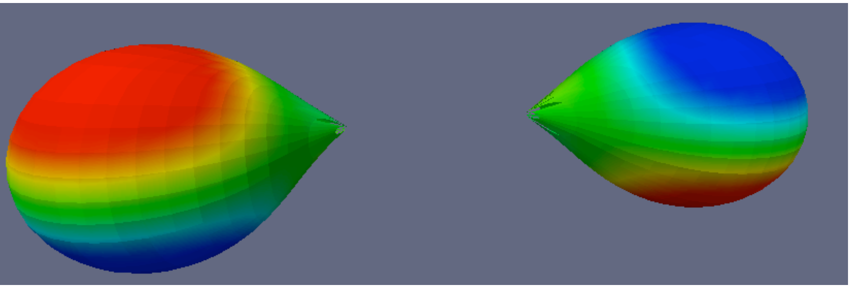
ω=(0.747-i 0.178)/2*M*

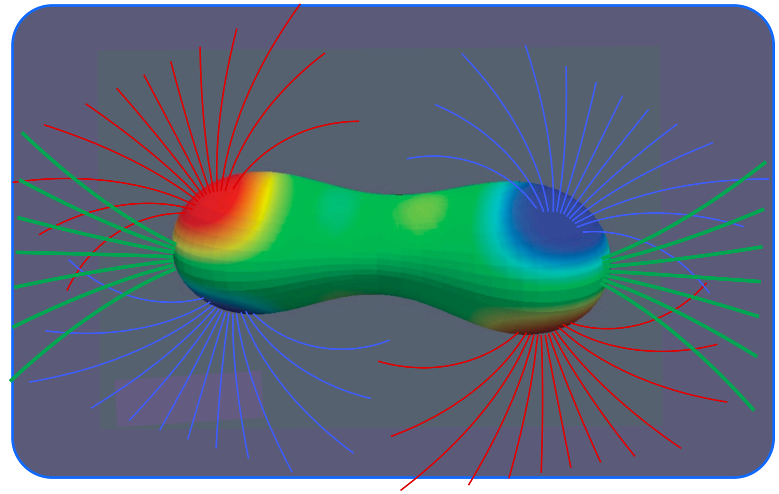
[Chandrasekhar & Detweiler]



- Vortex transitions smoothly into gravitational wave trough
- Slow slippage of vortex lines relative to vortex

Mergers of Spinning Holes
Simulations by Cornell/Caltech/CITA Group





Tendex lines

Vortex lines

- *For Relativists: On event horizon:*

$$\Psi_2 = \mathcal{E}_{NN} + i\mathcal{B}_{NN}$$

$$\mathcal{K} = \mathcal{R} + i\chi = -\Psi_2 + \mu\rho - \lambda\sigma$$

Complex curvature

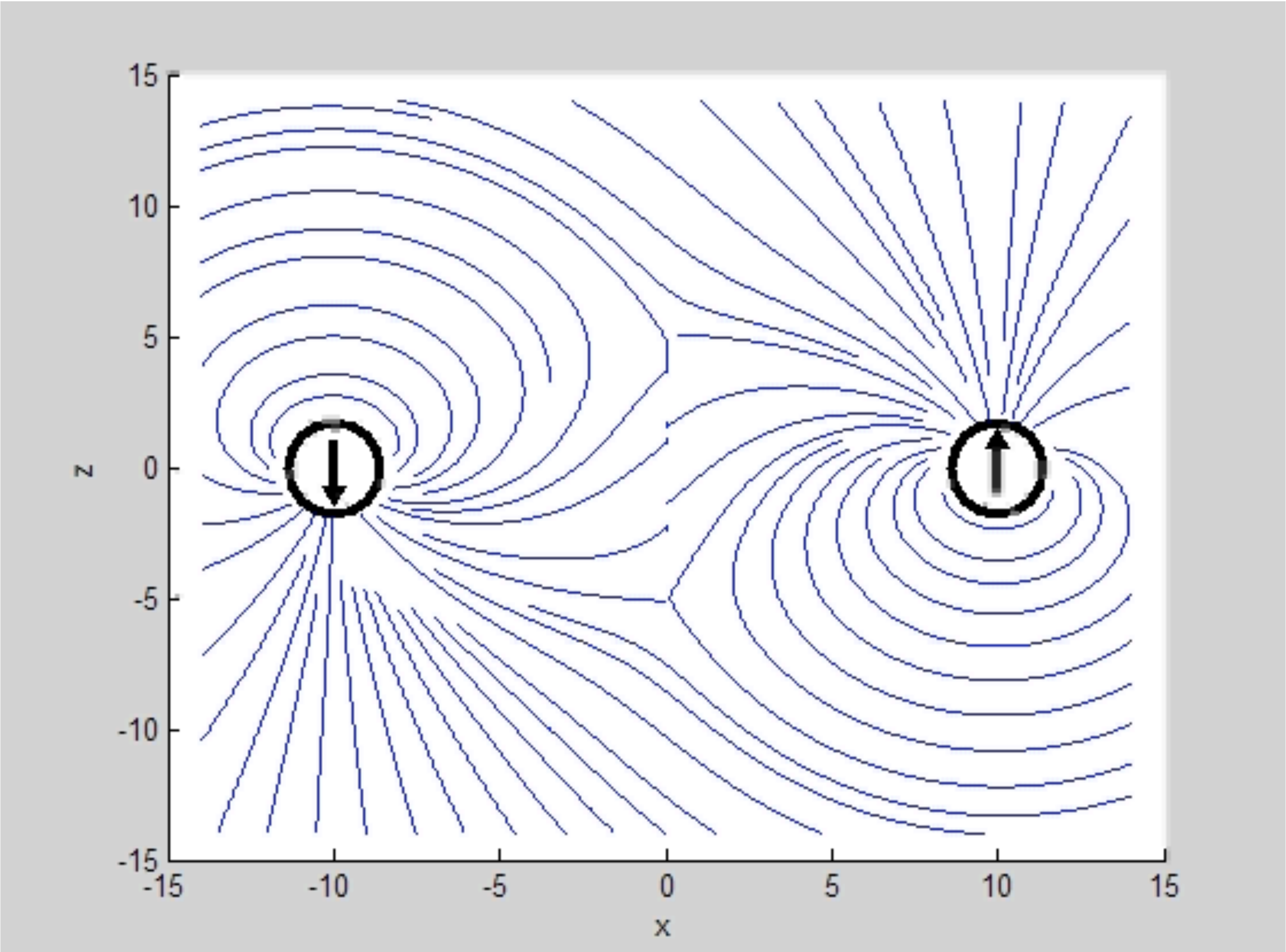
numerically small

Head-On Collision with Transverse Spins

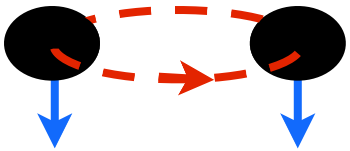
- Keith Matthews, Geoffrey Lovelace, Mark Scheel

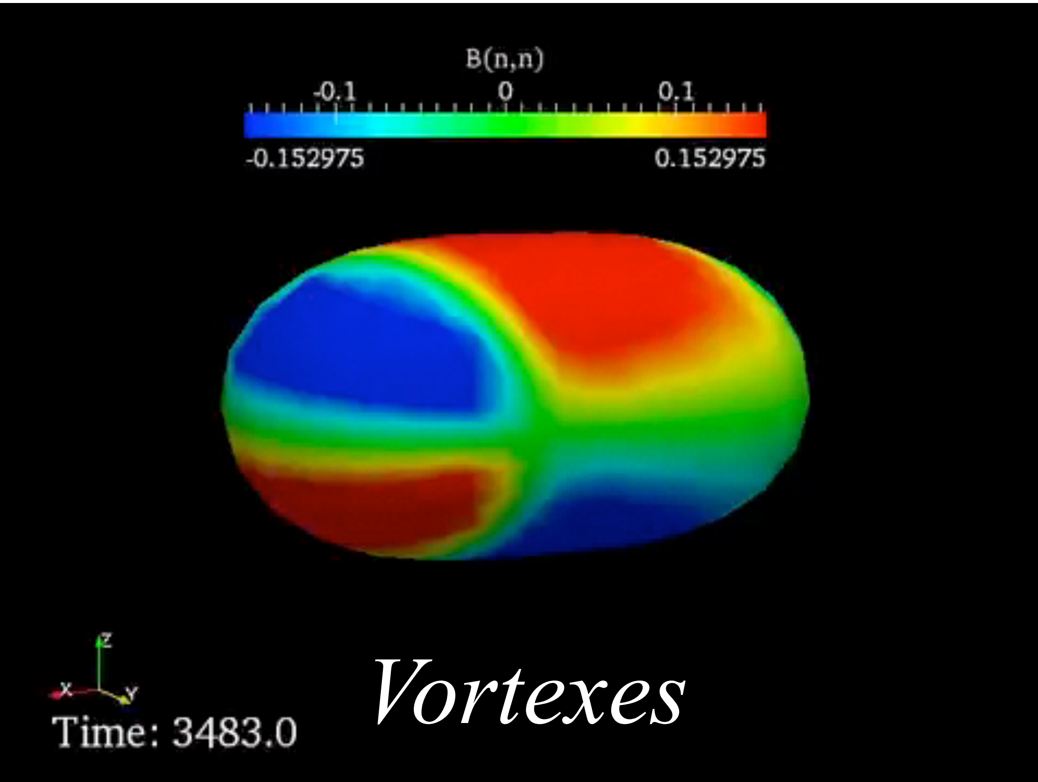
Head-On Collision with Transverse Spins

- Evolution of Negative Vorticity Vortex Lines During Inspiral
  - David Nichols



Anti-Aligned Inspiral and Merger

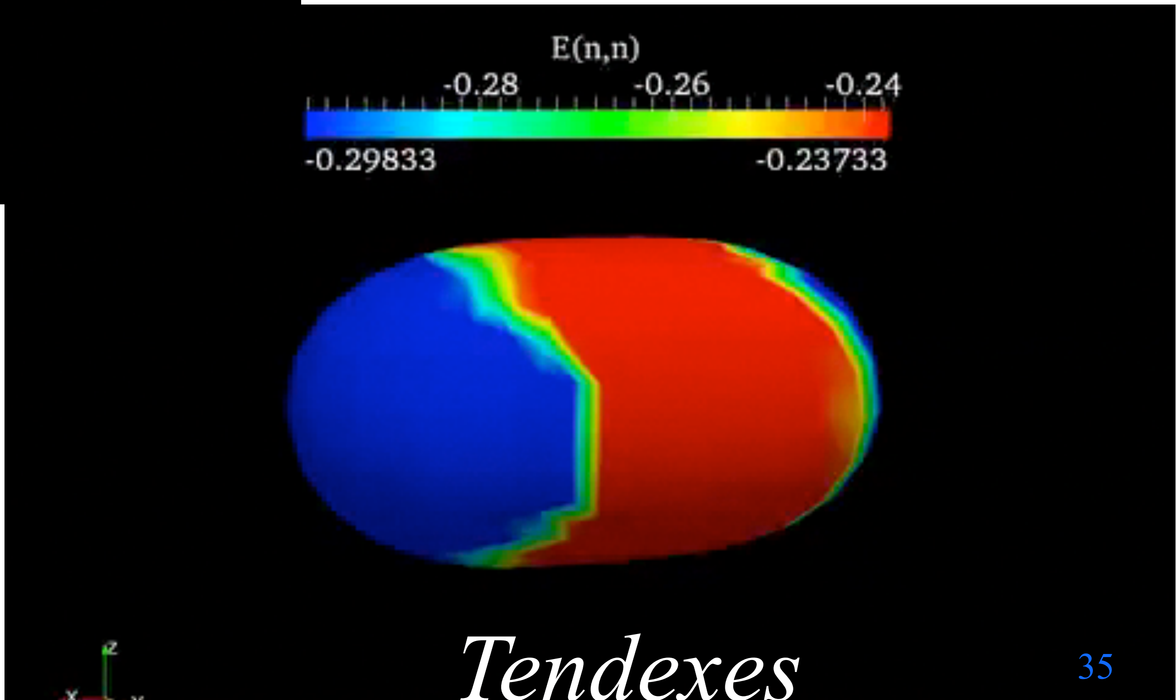




Spin 0.95

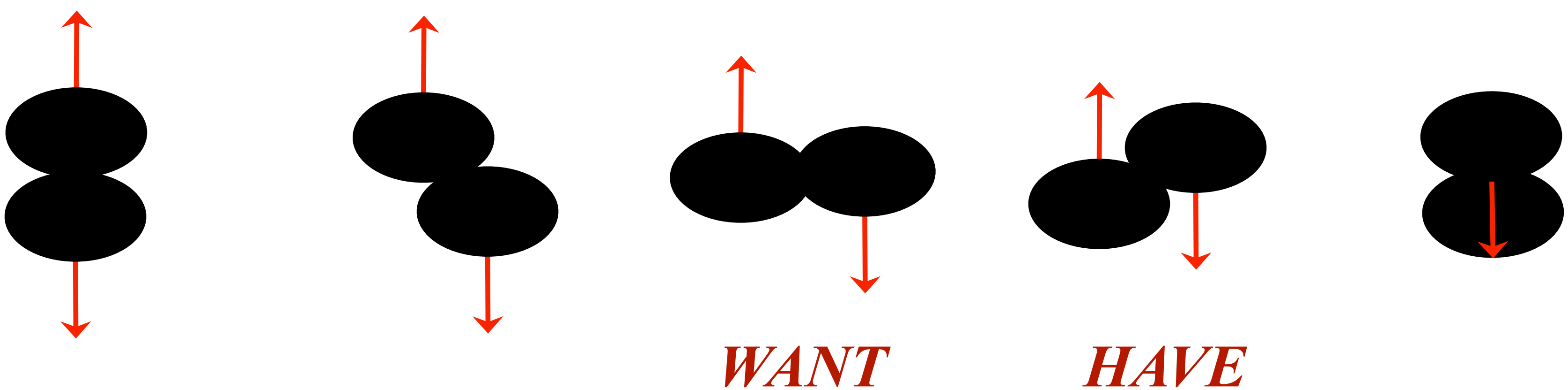
- Lovelace, Scheel, Szilagyi

Common Apparent Horizon just after merger



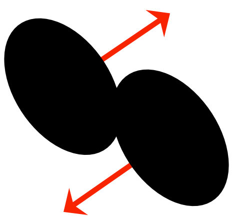
Extreme-Kick Configuration

Looking Down from Above

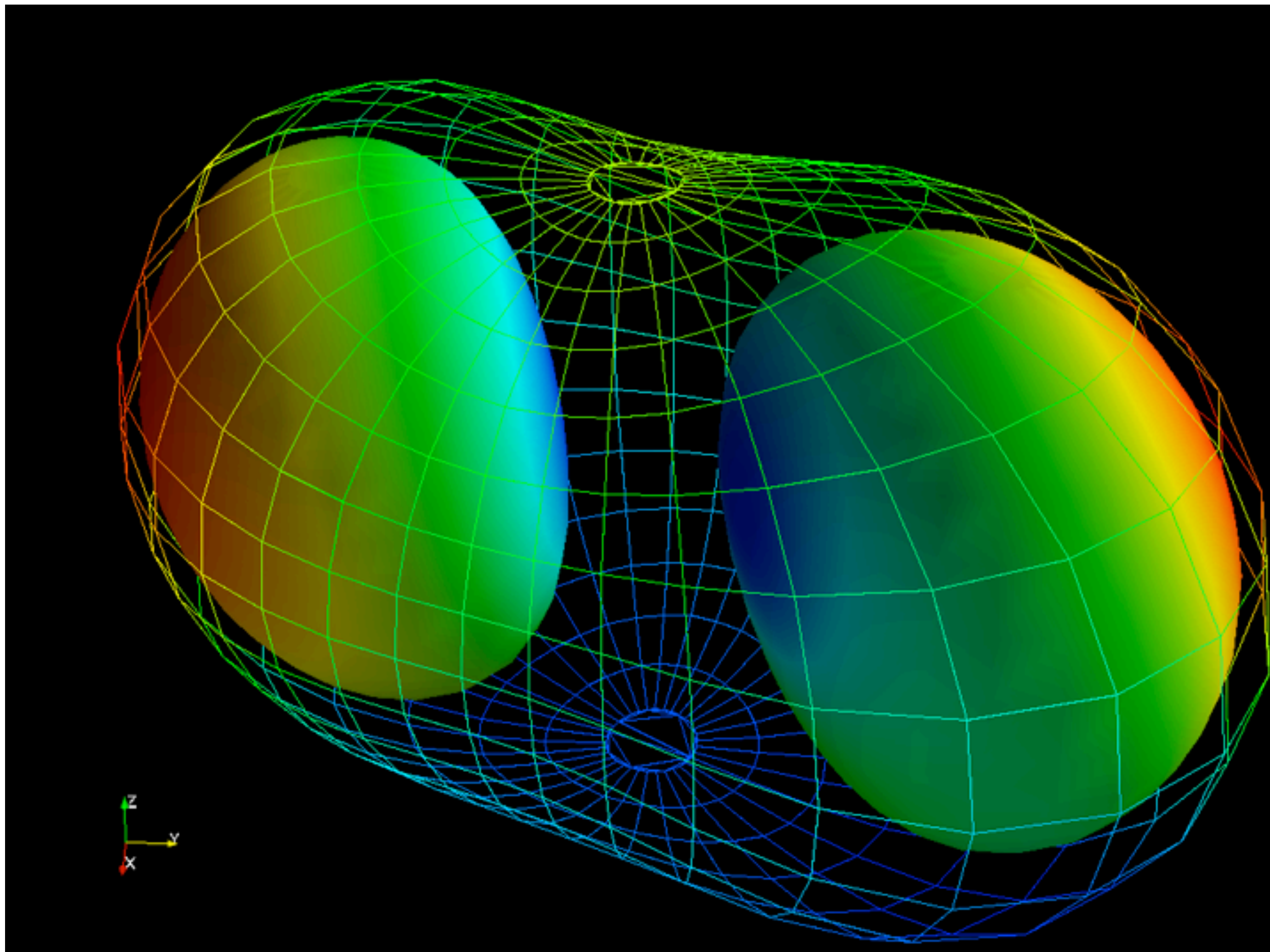


“Common” Apparent Horizon Surrounding Initial Holes’ MOTS

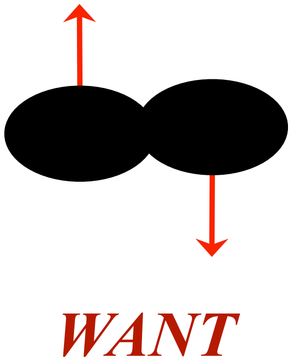
- Rob Owen [Cornell]

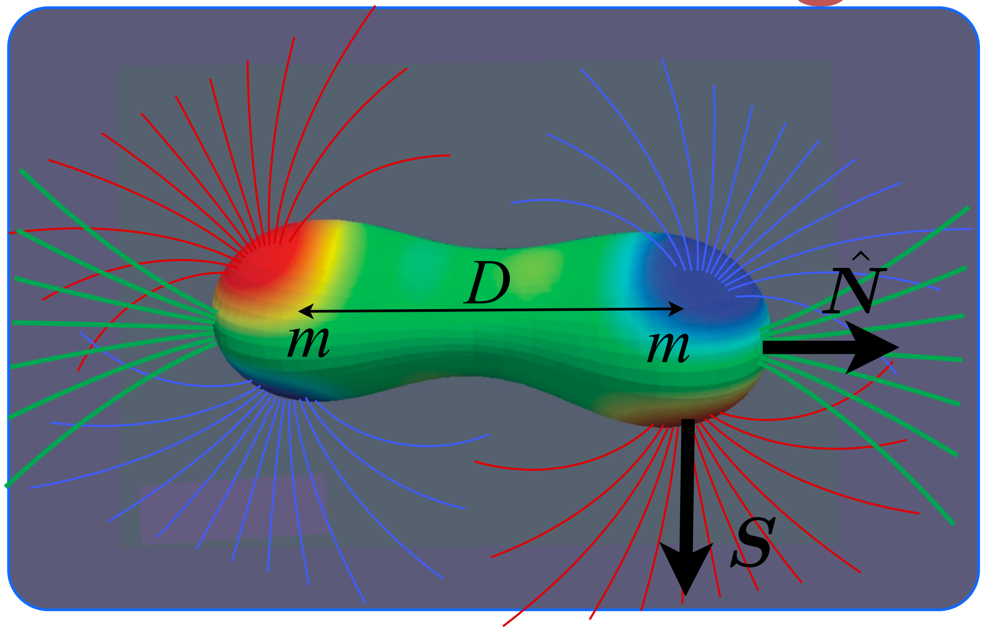


HAVE



Extreme-Kick Configuration



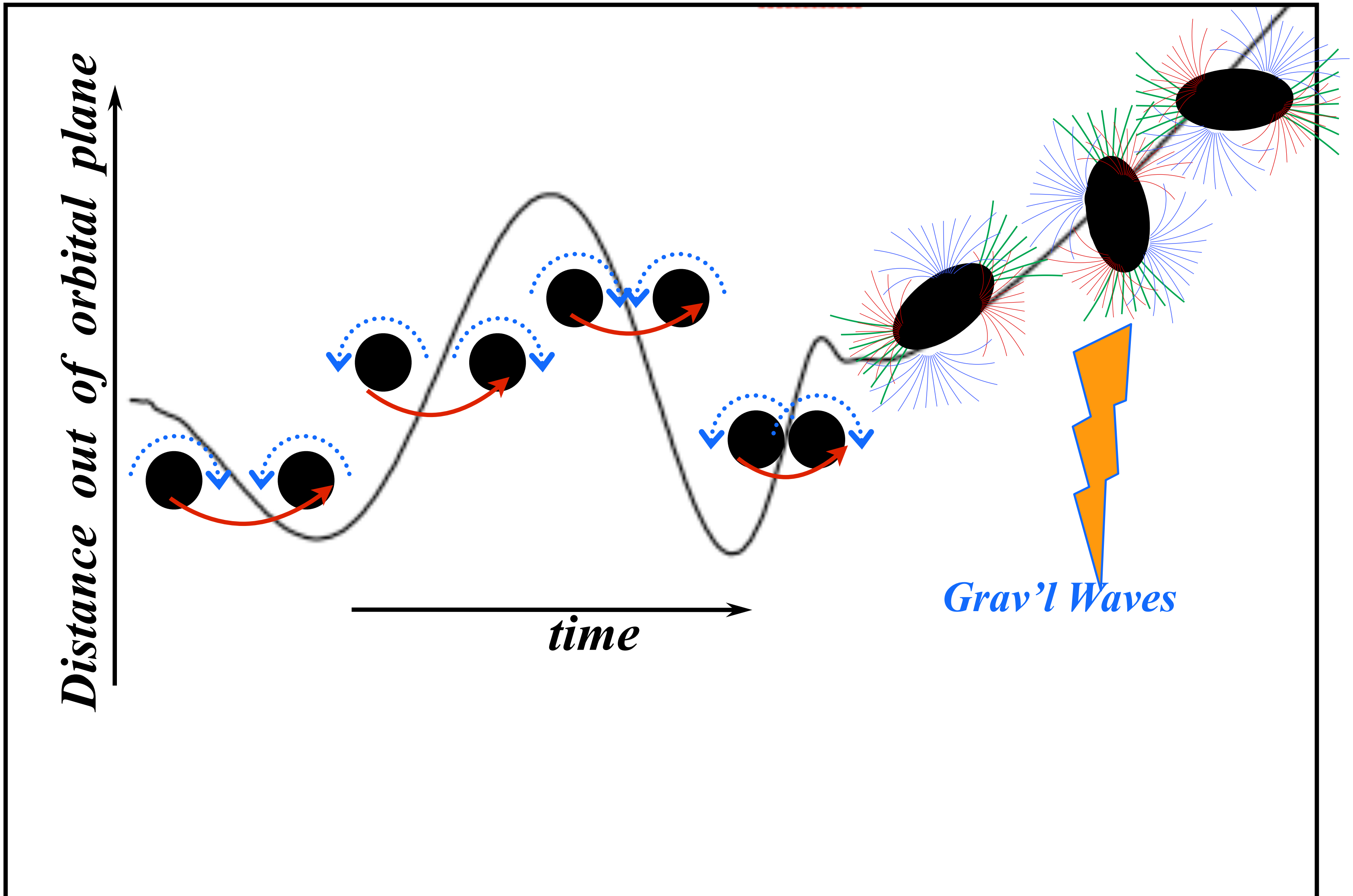


- Vortexes (**S**) rotate at Ω_V, Tendexes (**N**) at Ω_T
- Mass quadrupole GW's beat against current quadrupole GWs

$$\frac{d\boldsymbol{P}}{dt} = -\frac{16}{45}\dddot{\boldsymbol{\mathcal{I}}} \overset{\times}{:} \dddot{\boldsymbol{\mathcal{S}}}$$

$$= -\frac{64}{45} m^2 D^3 S\, \Omega_T^3 (\Omega_T + \Omega_V)^3 \hat{\boldsymbol{N}} \times \boldsymbol{S}$$

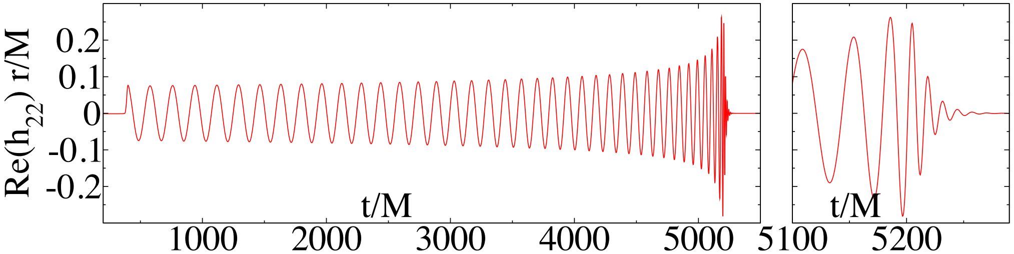
Extreme Kick



Unreasonable Simplicity of Waveforms

Unreasonable Effectiveness of PN Theory

What I expected



What simulations show

- Waveform simplicity:
  - » gravitational waves are produced by dynamics of tendexes and vortexes
  - » that dynamics appears to be remarkably simple -
  - » perhaps because the near zone is so thin (from r~2M to r~4M)
- Effectiveness of PN Theory (and Close limit Approximation) [Will's Talk]
  - » perhaps because these approximations capture well the dynamics of the tendexes and vortexes

Conclusion

- Black Holes show an amazing richness of physics
- Numerical Relativity has become a powerful tool
- Gravitational Waves will bring this rich physics into the realm of observations
- A new golden age of black hole research