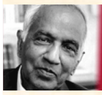
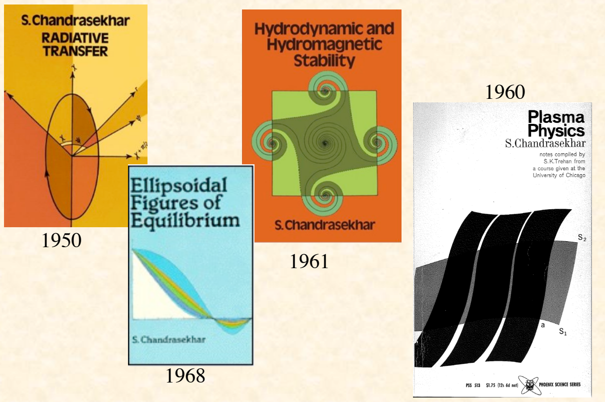


CHANDRASEKHAR CENTENNIAL SYMPOSIUM
OCTOBER 15, 16 & 17, 2010

Astrophysical Magnetohydrodynamics

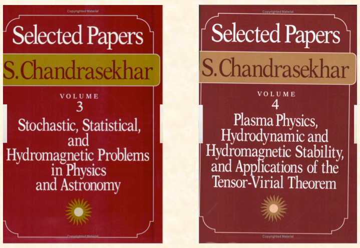
Jim Stone
Princeton University

My introduction to Chandrasekhar: his books.



Chandrasekhar's papers on MHD follow several themes.

1. Statistical properties of turbulence
2. Problems in astrophysical MHD
3. Plasma physics
4. HD and MHD instabilities



Outline of this talk.

1. Some elementary MHD
2. *Stars*: magnetoconvection.
3. *Accretion disks*: magnetorotational instability.
4. *Galaxies*: MHD turbulence in the ISM.
5. *Clusters*: effects of kinetic MHD.

The themes of Chandrasekhar's work:
- turbulence
- instabilities
- basic plasma processes,

will permeate all of these topics.

Chandrasekhar on astrophysical MHD, c1957.

It is clear we are very far from an adequate characterization of cosmic magnetic fields.

"On Cosmic Magnetic Fields", *Proceedings of the National Academy of Sciences* 43, no. 1 (1957) 24-27.

Much progress has been made since 1957…

…but much remains to be done.

1. Some elementary MHD

Ideal MHD = Euler equations + Maxwell equations + infinite conductivity.
Appropriate for highly collisional plasmas at low frequencies.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot [\rho \mathbf{v}\mathbf{v} - \mathbf{B}\mathbf{B} + P^*] = 0$$

$$\frac{\partial E}{\partial t} + \nabla \cdot [(E + P^*)\mathbf{v} - \mathbf{B}(\mathbf{B}\cdot\mathbf{v})] = 0$$

$$\frac{\partial B}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{B}) = 0$$

$E = \rho v^2/2 + e + B^2/2$ is total energy
$P^* = P + B^2/2$ is total pressure (gas + magnetic)

Warning: used units so that μ=1

Can be written in a compact form (in 1D): $\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} = 0$

Rewrite as: $\boxed{\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial \mathbf{U}}\frac{\partial \mathbf{U}}{\partial x} = 0}$ a nonlinear hyperbolic PDE.

Eigenvalues of Jacobian $\frac{\partial \mathbf{F}}{\partial \mathbf{U}}$ are characteristic speeds in MHD.

MHD waves.

Dispersion relation for plane waves in an isotropic, homogeneous medium:

$$\boxed{[\omega^2 - (\mathbf{k}\cdot\mathbf{v}_A)^2][\omega^4 - \omega^2 k^2 (v_A^2 + C^2) + k^2 C^2 (\mathbf{k}\cdot\mathbf{v}_A)^2] = 0}$$

Where $\mathbf{v}_A = \frac{\mathbf{B}}{\sqrt{4\pi\rho}}$ is the Alfven speed

$C^2 = \gamma P_0/\rho_0$ is the sound speed

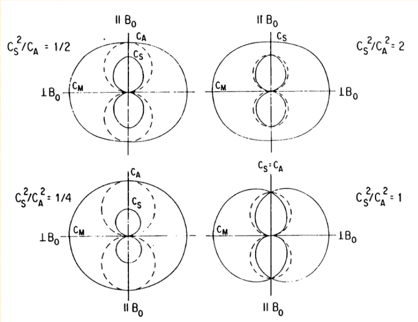
There are three modes (compared to one in hydrodynamics!):

Alfven wave propagates at V_A

Slow and fast magnetosonic waves propagating at C_s and C_f

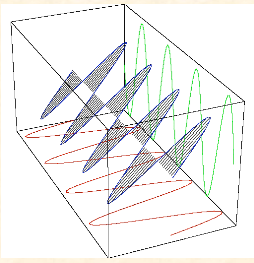
The rich dynamics of MHD results in part from the complexity of MHD waves.

Phase velocities of MHD waves: Friedrichs diagrams.



Note for in some cases, modes are degenerate. Eigenvalues of linearized MHD equations are not always linearly independent. MHD equations are not *strictly hyperbolic*.

Circularly polarized Alfven waves.

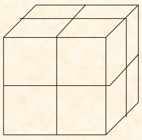


Since Alfven waves involve transverse motions, they can be polarized.

Circularly polarized waves are the sum of two linear polarizations with fixed phase shift.

Even in 1D, MHD requires 2 transverse velocity components in order to capture all MHD wave modes.

Numerical methods: finite-volume discretization.



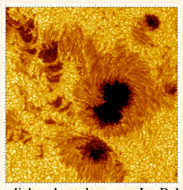
1. Discretize space into cells
   $\mathbf{x} \rightarrow (x_i, y_j, z_k)$
2. Discretize the continuous variables
   $\rho(\mathbf{x}, t) \rightarrow \rho^n_{i,j,k}$ where $\rho^n_{i,j,k} = \int \rho(\mathbf{x}, t^n) dV / \int dV$
3. Difference the conservation laws $\frac{\partial \mathbf{U}}{\partial t} = -\nabla \cdot \mathbf{F}(\mathbf{U})$
   as $\mathbf{U}_i^{n+1} - \mathbf{U}_i^n = -\frac{\Delta t}{\Delta x}\left(\mathbf{F}_{i+1/2}^{n+1/2} - \mathbf{F}_{i-1/2}^{n+1/2}\right)$

Higher-order Godunov schemes based on unsplit integrators, PPM reconstruction, and Constrained Transport are now popular.

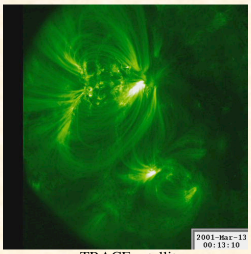
Mass, momentum, energy, and magnetic flux conserved to machine precision.

1. Solar magnetoconvection.

Observations of the outer layers of the sun provide the best evidence for importance of MHD.



Swedish solar telescope, La Palma



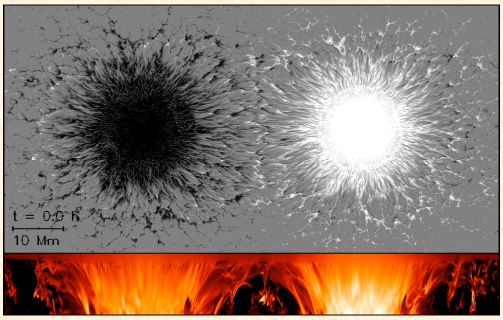
TRACE satellite

Two important areas of study:

1. Formation and structure of sunspots (small scale dynamo)
2. Solar cycle (large-scale dynamo)

Modeling sunspots

Direct numerical simulations of magnetoconvection, with realistic radiative transfer, can reproduce the details of observed sunspots.



B_z

B^2

Rempel et al., NCAR

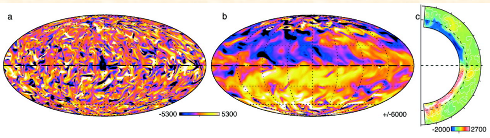
Solar dynamo

Sun's dipole field thought to originate in *tachocline*: region of shear between radiative core and convective envelope.

Global simulations of magnetoconvection still fail to fully explain:

- Origin of differential rotation in convective zone
- Origin of cyclic dipole field

Meisch et al.



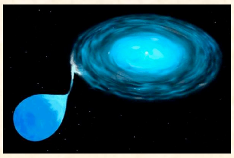
B_ϕ in convective zone $\quad$ B_ϕ in tachocline $\quad$ $\langle B_\phi \rangle$

Unlikely to understand stellar dynamos without understanding solar dynamo first.

2. The MRI in accretion disks.

Accretion disks are ubiquitous:

- Protostars
- Close binaries
- Active galactic nuclei



Astrophysics goal: to understand accretion disk structure and evolution as thoroughly as we understand stellar structure and evolution (luminosity, spectra, variability, etc.)

Accretion can only occur with outward angular momentum transport.

- microscopic viscosity too small
- anomalous (turbulent?) viscosity required

What drives turbulence?

Magnetorotational Instability (MRI)

1. Start with the equations of MHD for a rotationally supported disk.
2. Look for solutions of the form $\exp i(\mathbf{k}\cdot\mathbf{r} - \omega t)$ (local WKB analysis).

Simplifications. Consider only:

1. incompressible, axisymmetric perturbations
2. vertical field: $\mathbf{B} = (0, 0, B_z)$
3. ideal MHD.

Resulting dispersion relation:

$$\boxed{\omega^4 - \omega^2[\kappa^2 + 2(\mathbf{k}\cdot\mathbf{V}_A)^2] + (\mathbf{k}\cdot\mathbf{V}_A)^2\left((\mathbf{k}\cdot\mathbf{V}_A)^2 + \frac{d\Omega^2}{d\ln r}\right) = 0}$$

$\kappa^2 = \frac{1}{R^3}\frac{d(R^4\Omega^2)}{dR}$ (epicyclic frequency) $\quad V_A^2 = B^2/4\pi\rho$ (Alfven velocity)

Instability ($\omega^2 < 0$): $\boxed{(\mathbf{k}\cdot\mathbf{V}_A)^2 < -\frac{d\Omega^2}{d\ln r}}$

Historical development of MRI

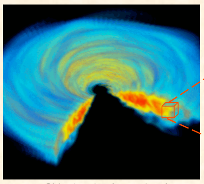
Velikov (1959) - first discovery of instability in rotating plasma expt.
Chandrasekhar (1960) - global instability in magnetized Couette flow
Fricke (1969) - instability in differentially rotating stars
Safronov (1972) - argued instability would not be important in disks
Balbus & Hawley (1991) - rediscovery of instability, first recognition of importance in accretion disks.

From BH (1991):

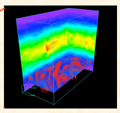
The essence of our result is that the *Rayleigh instability criterion of a negative radial gradient in specific angular momentum is laregly irrelevant to gaseous astrophysical disks. Instead the combination of a negative angular velocity radial gradient with almost any small seed field will lead to dynamical instability.*

While all of this may seem surprising, a related process was studied long ago by Chandrasekhar (1960). He considered the *global* stability of a vertically magnetized column of incompressible fluid undergoing Couette flow, and for a vanishingly small field found precisely the same instability criterion described above. In his paper, Chandrasekhar (1960) noted explicitly, and with some surprise, the nonemergence of the Rayleigh criterion in the vanishing field limit.

Nonlinear saturation of MRI studied with both *local* ("shearing-box") and *global* simulations.



Global simulation

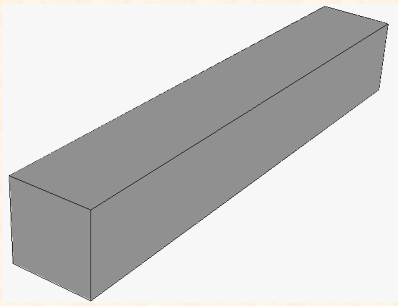


Local simulation

Local simulation: expand MHD equations in a frame orbiting at the local angular velocity Ω_0. Introduces Coriolis force and tidal gravity as source terms.

2. Local simulation of MRI in 3D

In 3D, MRI drives sustained turbulence.

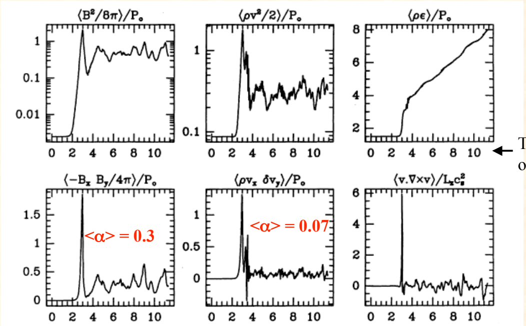


Athena MHD code
128 x 256 x 128 Grid
$\beta = 100$, toroidal field
orbits 4-20

Animation of angular velocity fluctuations: $\delta V_\phi = V_\phi - V_{Kep}$

Significance of the MRI: vigorous angular momentum transport.

Time-evolution of volume-averaged quantities:



Also note: Sustained amplification of B indicates dynamo action

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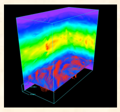
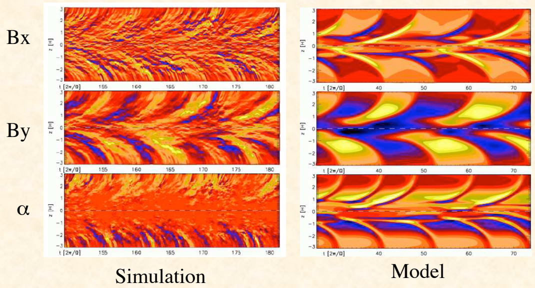
Time evolution of horizontally averaged quantities in *stratified* disks reveal buoyant rise of flux, and a cyclic dynamo.





Gressel (2010) has modeled "butterfly diagram" as mean-field dynamo.

Numerical simulations have established:

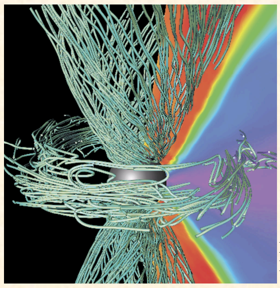
- MRI saturates as MHD turbulence.
- Maxwell stress dominates Reynolds stress.
- Significant transport possible ($\alpha \sim 0.3$ for vertical fields).
- Stress is local, correlation length < H.
- Turbulence amplifies field; drives a MHD dynamo.
- Power spectrum is anisotropic, most energy on largest scales.

Important questions remain:

- Is energy dissipated primarily in ions or electrons or both?
- Importance of internal versus external torques (winds)?
- Is super-Eddington accretion possible?

Global models of disks & jets in full GR.

de Villiers, Hawley, & Krolik
McKinney et al.
Gammie et al.

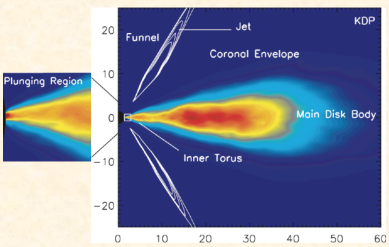


Evolve GRMHD equations in fixed spacetime.

Study properties of flow near event horizon, production of relativistic jets.

Magnetic field lines + log(density)
Hawley & Krolik

Typical time-averaged flow structure



Production of relativisitic jet requires

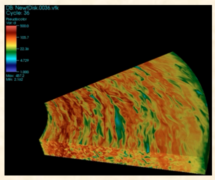
- Net vertical field in inner region
- Rotating black hole

Non-zero torque at ISCO for geometrically thick disks.

There is little doubt the MRI is the mechanism for angular momentum transport in most disks.
(gravitational instabilities important in massive disks).

Future work is focused on:

- Improved physics
  - Non-ideal MHD effects in protoplanetary disks
  - Radiation MHD effects in BH accretion disks
- Global models with radiation transport that can be compared directly to observations.



Global simulations with the same numerical resolution per H are now possible.

3. MHD Turbulence in the ISM of galaxies.

The interstellar medium (ISM) of galaxies is observed to be magnetized.

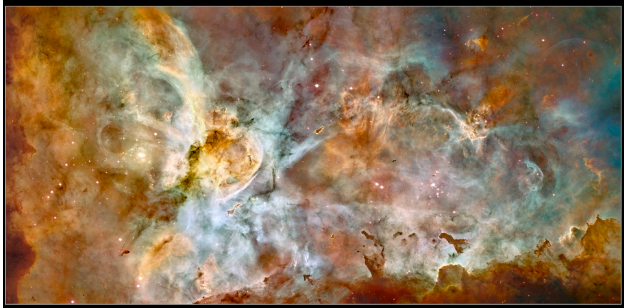
Equipartition fields:
$B^2/8\pi = P_{gas} = P_{CR}$

Synchrotron emission and polarization vectors in M51

Moreover, ISM of galaxies is turbulent

- Highly compressible: $\sigma_v >> C_s$
- Strongly magnetized: $C_A \sim \sigma_v >> C_s$

Carina Nebula



Hubble Heritage

Chandrasekhar on turbulence theory.

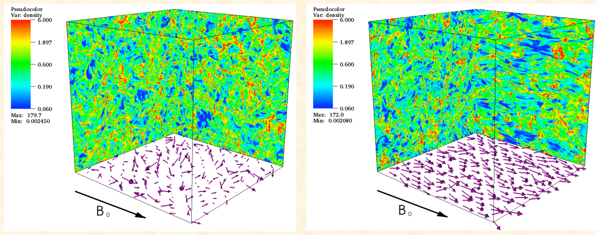
We cannot construct a rational physical theory without an adequate base of physical knowledge. It would therefore seem to me that we cannot expect to incorporate the concept of turbulence in astrophysical theories in any essential manner without a basic physical theory of the phenomenon of turbulence itself.

S. Chandrasekhar, "Turbulence--A Physical Theory of Astrophysical Interest (Henry Norris Russell Lecture)", The Astrophysical Journal 110, no. 3 (1949): 329-39.

Fortunately, theory of energy cascades in strong MHD turbulence has progressed substantially in the last few decades.
Goldreich-Sridhar (1995)
Boldyrev (2006)
Beresynyak & Lazarian (2010)

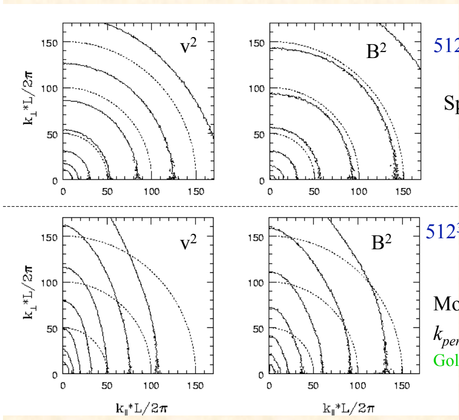
Direct MHD simulations of driven *supersonic* turbulence reveal statistics of density and magnetic field.

both 512^3 grid



M ~ 7,
$M_A \sim 1/2$ (strong field case)

2D power spectra are *anisotropic* with strong B



512^3 weak B (β=1)

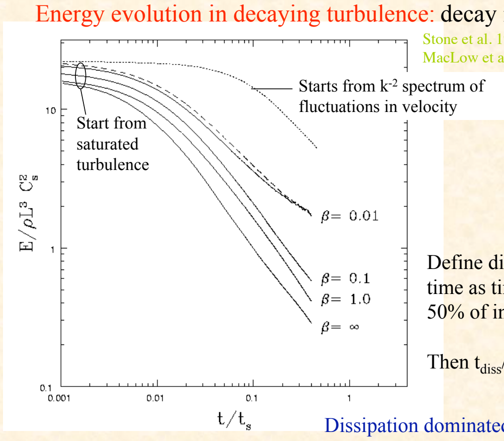
Spectra isotropic

512^3 strong B (β=0.01)

More power along k_{perp} than k_{par}
Goldreich & Sridhar 1995

Energy evolution in decaying turbulence: decay is rapid

Stone et al. 1998
MacLow et al 1998

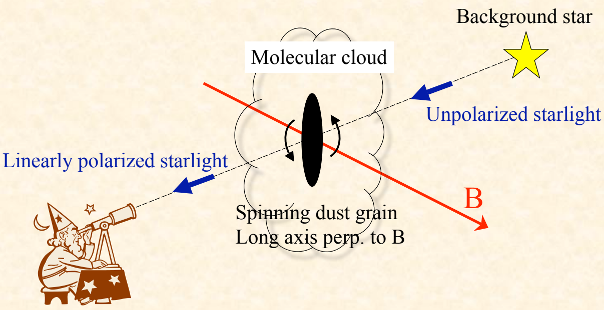


Define dissipation time as time to lose 50% of initial E

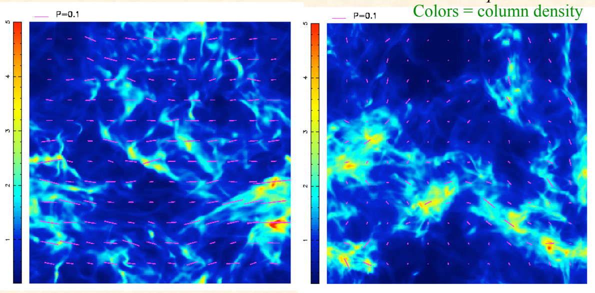
Then $t_{diss}/t_{cross} = 0.8$

Dissipation dominated by shocks

Magnetic field in molecular clouds probed by polarization of starlight



Scatter in polarization angle $\delta\phi$ depends on magnetic field strength in plane-of-sky B_p.



Strong field – scatter small　　Weak field – scatter large

Results show $B_p = 0.5(4\pi\ \rho)^{1/2}\ \delta v\ /\ \delta\phi$

"Chandrasekhar-Fermi method" is now routinely used to measure B_p.

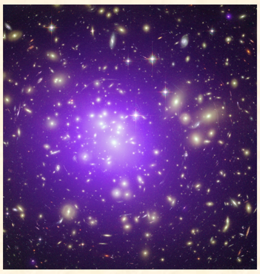
4. Kinetic MHD effects in clusters of galaxies.

Observations indicate the X-ray emitting plasma in clusters of galaxies is in the *kinetic MHD* (long mean-free path) regime:

$\lambda \ll L$, $\lambda \gg \rho$ (gyro-radius)

For example: Abell 1689

Purple=Chandra X-ray image
Yellow=Hubble optical image



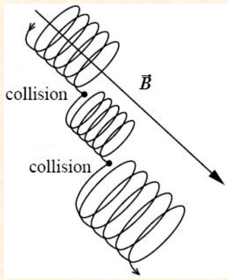
T ~ 4.5 keV, n ~ 10^{-3}-10^{-4} cm^{-3}, B~ 1μG implies $\lambda_{mfp} \sim 0.1 R_V$
$\rho \sim 10^8$ cm

What are the equations of motion in the kinetic MHD regime?

Key property:
anisotropic transport coefficients.

When *electron* mean free path is much longer than electron gyro-radius: **anisotropic heat conduction**.

When *ion* mean free path is much longer than ion gyro-radius: **anisotropic viscosity**.



Simplest description is MHD with anisotropic thermal conduction and viscosity (Braginskii 1965).

Kinetic MHD: qualitatively different.

Anisotropic transport coefficients produce *qualitative* changes to the dynamics:

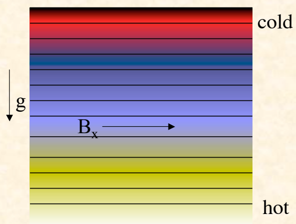
With anisotropic heat conduction, the convective instability criterion becomes dT/dz < 0 (Balbus 2000) (magneto-thermal instability, MTI)

Gradients of temperature, not entropy, determine stability.
Much like MRI, where gradients of angular velocity, not angular momentum, determine stability.

Magneto-thermal instability (MTI).

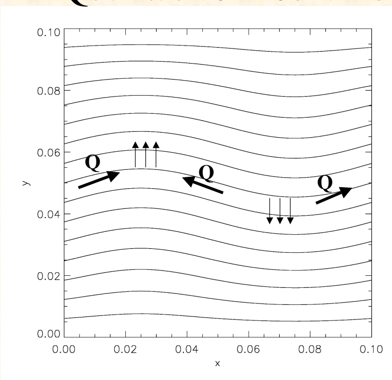
Consider a plane-parallel stratified atmosphere in a uniform gravitational field.

Temperature Profile



- Convectively Stable (dS/dz > 0)
- MTI unstable (dT/dz < 0)
- Ideal MHD + Anisotropic Heat Conduction
- g = constant
- BC's: adiabatic at y-boundary, periodic in x

Qualitative Mechanism



cold

Increasing temperature

hot

Field Line Plot of Single Mode Perturbation

Convection in stratified atmosphere is vigorous, and sustained in 3D (Parrish & Stone 2007)

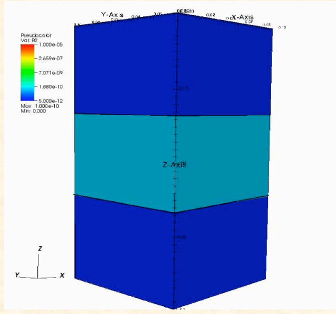
Vertical equilibrium with unstable layer between fixed-T boundaries.

$\mathbf{B} = (B_o \sin(z), B_o \cos(z), 0)$

128x128x256 grid

Animation of B^2

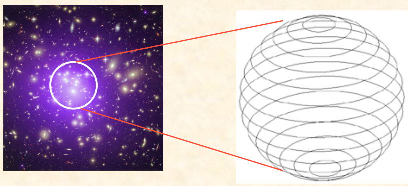
Initial entropy profile is *stable*. Convection would not occur if not for MTI.



MTI in an X-ray cluster.

Initial condition:

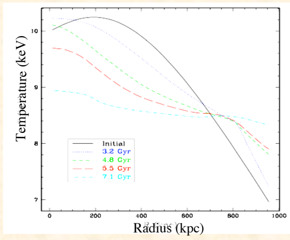
- spherical hydrostatic equilibrium
- dark matter potential
- initially ***toroidal*** field (parallel to isotherms)



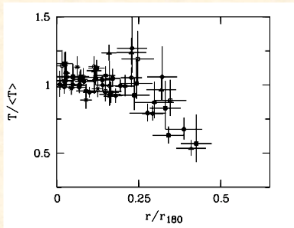
Evolve with 3D MHD and anisotropic thermal conduction.
(Parrish & Stone 2008)

MTI in an X-ray cluster.

Growth of MTI causes redistribution of T profile, and significant field amplification in a Hubble time.



Spherically averaged T-profile



Observed T-profile of Hydra cluster

MTI + feedback + turbulence from mergers of substructure may all be important.

Summary

Magnetohydrodynamics is now understood to be fundamental for understanding many basic problems in astrophysics, *e.g. angular momentum transport in accretion disks*.

Many frontiers exist, e.g. going beyond MHD, to kinetic MHD.

Computational methods are now the primary tool for the investigation of nonlinear, time-dependent MHD.

What would Chandrasekhar think of modern, computational methods?

Chandrasekhar's contributions to astrophysical MHD and plasma physics endure. His work on the MRI was before its time.