Gravitational waves from perturbed stars

Valeria Ferrari
Dipartimento di Fisica,   Sapienza   Università di Roma

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The theory of perturbations of non rotating stars was first developed by Prof. Thorne and collaborators in the late Sixthies

Thorne & Campolattaro, A. 1967 ApJ ***149****, 591*
Thorne & Campolattaro, A. 1967 ApJ ***152****, 673*
Thorne 1969 ApJ ***158*** *1*
Thorne 1969 ApJ ***158*** *997*
Price & Thorne 1969 ApJ ***155****, 163*
Campolattaro, Thorne 1970 ApJ ***159****, 847*
Ipser, Thorne 1973 ApJ ***181****, 181*

Chandra wanted to attack the problem from a different point of view, in analogy with the theory of black hole perturbations

black hole perturbations	⟷	stellar perturbations

Some basic results of the theory of black hole perturbations:

Perturbations of Schwarzschild black holes

$$\frac{d^2 Z_\ell^{\pm}}{dr_*^2} + \left[\omega^2 - V_\ell(r)\right] Z_\ell^{-} = 0 \qquad r_* = r + 2M log(r/2M - 1)$$

$$V_\ell^{-}(r) = \frac{1}{r^3}\left(1 - \frac{2M}{r}\right)\left[\ell(\ell+1)r - 6M\right]$$

Regge&Wheeler 1957

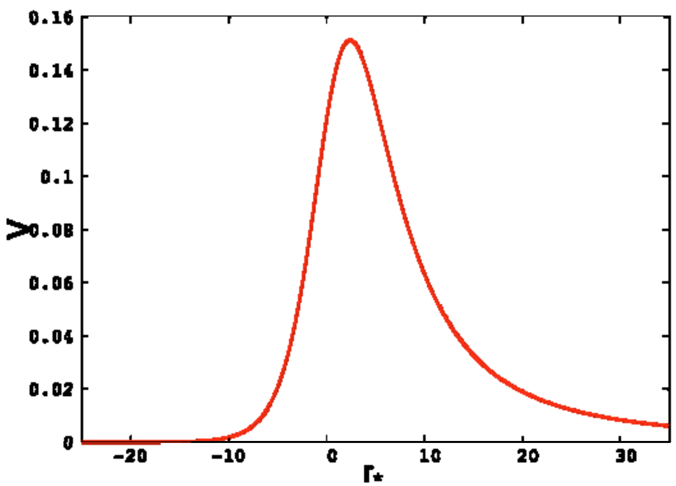
$$V_\ell^{+}(r) = \frac{2(r-2M)}{r^4(nr+3M)^2}\left[n^2(n+1)r^3 + 3Mn^2r^2 + 9M^2nr + 9M^3\right]$$

F. Zerilli 1970

$$n = (\ell+1)(\ell-2)/2$$

$Z_\ell^{\pm}$ - axial (odd) perturbations

+ polar (even) perturbations



A wave equation also for the perturbations of Kerr black holes

$$\Delta R_{lm,rr} + 2(s+1)(r-M)R_{lm,r} + V(\omega,r)R_{lm} = 0$$
$$\Delta = r^2 - 2Mr + a^2$$

S.Teukolsky 1972 Phys. Rev. Lett. 29, 1114
S.Teukolsky 1973 Ap. J. 185, 635

$$\begin{aligned} V(\omega,r) &= \frac{1}{\Delta}\left[(r^2+a^2)^2\omega^2 - 4aMrm\omega + a^2m^2 + \right. \\ &+ \left. 2is(am(r-M) - M\omega(r^2-a^2))\right] + \left[2is\omega r - a^2\omega^2 - A_{lm}\right] \end{aligned}$$

the potential is complex and depends on m and on frequency

$$\psi_s(t,r,\theta\varphi) = \frac{1}{2\pi}\int e^{-i\omega t}\sum_{l=|s|}^{\infty}\sum_{m=-l}^{l} e^{im\varphi}S_{lm}(\cos\theta)R_{lm}(r)d\omega$$

$S_{lm}(\cos\theta)$ satisfies the equations of the oblate spheroidal harmonics

s= is the spin-weight parameter, s=0, ± 1, ± 2,
for scalar, electromagnetic and gravitational perturbations

- Black hole perturbations are described by wave equations, with one-dimensional potential barrier generated by the spacetime curvature

- Black hole perturbations can be studied as a scattering problem.

- Standard methods used in quantum mechanics can be used to find the quasi-normal mode frequencies: they are the singularity of the scattering cross-section associated the wave equation

In quantum mechanics the equation which expresses the symmetry and unitarity of the S-matrix

$$|R|^2 + |T|^2 = 1$$

is an energy conservation law: if a wave of unitary amplitude is incident on one side of the potential barrier, it gives rise to a reflected and a transmitted wave such that the sum of the square of their amplitudes is still one.

This conservation law is a consequence of the constancy of the Wronskian of pairs of independent solutions of the Schroedinger equation.

Same is for black holes: the constancy of the Wronskian of two independent solutions of the black holes wave equation, allows to write the same equation relating the reflection and transmission coefficients of the potential barrier.

Energy conservation also governs phenomena involving gravitational waves emitted by perturbed black holes.

The reflection and absorption of incident gravitational waves by a perturbed black hole or by a perturbed star, become different aspects of the same basic theory.

This idea needed to be substantiated by facts: first of all we needed an equation expressing the conservation of energy for stellar perturbations.

Problem: black hole perturbations are described by wave equations, whereas the equations describing a perturbed star are a higher order system, in which the perturbation of the metric functions couple to the perturbations of the fluid.

working hard on the equations we were able to derive a vector in terms of the metric and fluid perturbations

$$\frac{\partial}{\partial x^\alpha} E^\alpha = 0, \qquad \alpha = (x^2 = r, x^3 = \vartheta)$$

by Gauss' theorem, the flux of E^α across a closed surface surrounding the star is a constant.

- Which is the physical meaning of this vector?
- How do we establish that it is the flux of gravitational energy which develops through the star and propagates outside?
- Is it related to a stress-energy pseudotensor of the gravitational field?

$$\frac{\partial}{\partial x^\nu} \left[\sqrt{-g}\,(T^{\mu\nu} + t^{\mu\nu})\right] = 0$$

Chandrasekhar S. and Ferrari V. 1990, Proc. R. Soc. Lond. **428**, 325
Chandrasekhar S. and Ferrari V. 1991, Proc. R. Soc. Lond. **435**, 645

We derived the equations describing the perturbations of a non rotating star in the same gauge used for Schwarzschild perturbations

remind:

polar (even) equations couple metric and fluid perturbations

axial (odd) equations describe pure spacetime perturbations

Some results obtained with the new approach

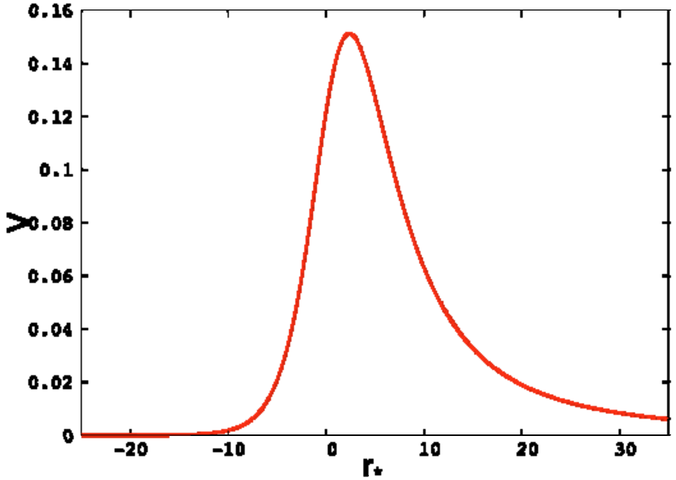
A wave equation for the axial perturbations

Chandrasekhar S. and Ferrari V. 1991, Proc. R. Soc. Lond. **434**, 449

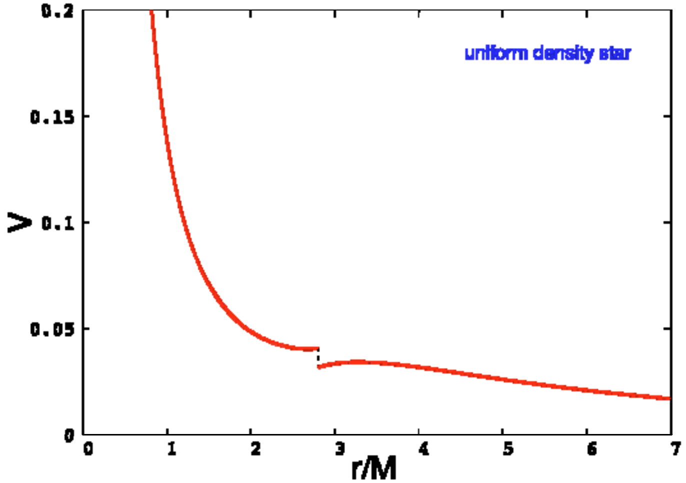
$$\frac{d^2 Z_\ell}{dr_*^2} + [\omega^2 - V_\ell(r)] Z_\ell = 0$$

$$V_\ell(r) = \frac{e^{2\nu}}{r^3}[\ell(\ell+1)r + r^3(\epsilon - p) - 6m(r)], \qquad \nu_{,r} = -\frac{p_{,r}}{\epsilon + p}$$

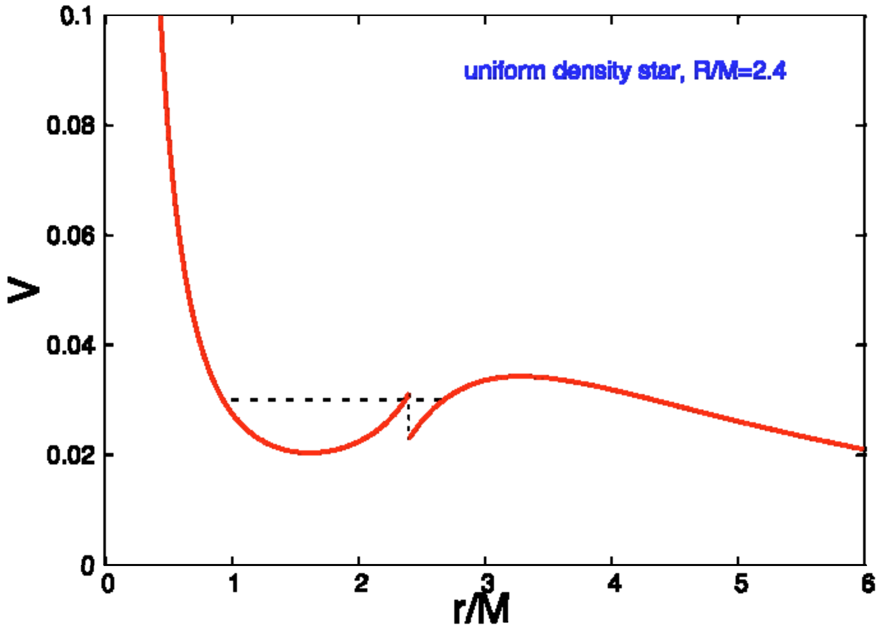
the potential barrier depends on how the energy-density and the pressure are distributed inside the star in its equilibrium configuration. For r > R it reduces to the Regge_Wheeler potential



black holes: scattering by a one-dimensional potential barrier



stars: scattering by a central potential



if we look for solutions that are regular at r=0 and behave as pure outgoing waves at infinity

$$Z_\ell \rightarrow e^{-i\omega r_*}, \qquad r_* \rightarrow \infty$$

we find modes which do not exists in Newtonian theory

- if the star is extremely compact, the potential in the interior is a well, and if this well is deep enough there can exist one or more *slowly damped QNMs (or s-modes)*
- another branch of modes are the *w- modes* they are associated to the scattering of GW- waves at the peaks of the barrier. They are *higly damped*

Chandrasekhar S. and Ferrari V. 1991, Proc. R. Soc. Lond. **434**, 449
Kokkotas K.S. 1994, MNRAS ***268***, 1015

Until very recently, the common belief was that *w- modes* are unlikely to be excited in astrophysical processes. However in 2005 it has been shown that, they are excited in the collapse of a neutron star to a black hole, just before the black hole forms

Baiotti L., Hawke I., Rezzolla L. and Schnetter E. 2005, *Phys. Rev. Lett.* **94**, 131101

The determination of the characteristic frequencies at which a star oscillates is central to the theory of stellar perturbation, since these frequencies appear to be encoded in various radiative processes

any initial perturbation will, after a transient, decay as a superposition of quasi normal modes

gravitational waves are emitted with the frequencies and damping times of these modes.

NORMAL MODES IN NEWTONIAN THEORY

- the star is set in oscillation by some unspecified external agent.
- After separating the variables, the hydrodynamical equations are manipulated in such a way that the quantity which is singled out to describe the perturbed star is the Lagrangian displacement

$$\vec{\xi}$$

- in its terms, the accompanying changes in the density, the pressure and the gravitational potential, can all be expressed uniquely.
- by assuming $$\xi^\mu = \xi^\mu(r)e^{i\omega t}$$

the linearized hydro-equations are reduced to a characteristic value problem for the frequency.

Our approach to the polar perturbations is different from that of Newtonian theory, since our emphasis is on the gravitational field rather than on the fluid behaviour.

The polar perturbations couple the perturbations of the metric with the fluid perturbations

however

the equations governing the metric variables can be decoupled from those for the hydrodynamical variables (in the same gauge as for Schwarzschild pert.)

once the equations governing the metric functions have been solved, the solution for the hydrodynamical variables follows *algebraically*, without any further ado.

Chandrasekhar S. and Ferrari V. 1991, Proc. R. Soc. Lond. **432**, 247
Chandrasekhar S., Ferrari V. and Winston R. 1991, Proc. R. Soc. Lond. **434**, 635
Chandrasekhar S. and Ferrari V. 1992, Proc. R. Soc. Lond. **437**, 133

Given an equilibrium configuration, for any assigned EOS, it is very easy to evaluate the QNM frequencies by integrating the equations for the metric perturbations inside and outside the star.

An interesting consequence of the theory

if the star is non rotating, axial and polar perturbations are described by two distinct sets of equations

Slowly rotating stars: the axial perturbations couple to the polar ones (and viceversa)

Chandrasekhar S. and Ferrari V. 1991, Proc. R. Soc. Lond. **433**, 423

$$\mathbf{Z}^{\mathbf{ax}} = \mathbf{Z}^{\mathbf{0}\ \mathbf{ax}} + \epsilon(\Omega)\mathbf{Z}^{\mathbf{1}\ \mathbf{ax}}$$

$$\sum_{\ell=2}^{\infty}\left\{\frac{d^2\mathbf{Z}_\ell^{\mathbf{0}}}{dr_*^2} + \omega^2\mathbf{Z}_\ell^{\mathbf{0}} - \frac{e^{2\nu}}{r^3}[\ell(\ell+1)r + r^3(\epsilon - p) - 6m(r)]\mathbf{Z}_\ell^{\mathbf{0}}\right\} C_{\ell+2}^{-\frac{3}{2}}(\mu) = 0$$

$$\sum_{\ell=2}^{\infty}\left\{\frac{d^2\mathbf{Z}_\ell^{\mathbf{1}}}{dr_*^2} + \sigma^2\mathbf{Z}_\ell^{\mathbf{1}} - \frac{e^{2\nu}}{r^3}[\ell(\ell+1)r + r^3(\epsilon - p) - 6m(r)]\mathbf{Z}_\ell^{\mathbf{1}}\right\} C_{\ell+2}^{-\frac{3}{2}}(\mu)$$

$$= \quad r e^{2\nu - 2\mu_2}(1-\mu^2)^2 \sum_{\ell=2}^{\infty} \mathbf{S}_\ell^{\mathbf{0}}(r,\mu)$$

$$\mathbf{S}_\ell^{\mathbf{0}} = \varpi_{,r}[2\mathbf{W}_\ell^{\mathbf{0}} + \mathbf{N}_\ell^{\mathbf{0}} + 5\mathbf{L}_\ell^{\mathbf{0}} + 2n\mathbf{V}_\ell^{\mathbf{0}}P_{\ell,\mu} + 2\mu\mathbf{V}_\ell^{\mathbf{0}}P_{\ell,\mu,\mu}] + 2\boldsymbol{\varpi}\mathbf{W}_\ell^{\mathbf{0}}(Q-1)\nu_{,r}P_{\ell,\mu},$$

functions describing the polar perturbations at zero order on the angular velocity

function responsible for the dragging of inertial frames

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$$= re^{2\nu-2\mu_2}(1-\mu^2)^2\sum_{\ell=2}^{\infty}\mathbf{S}^{\mathbf{0}}_{\ell}(r,\mu)$$

$$\mathbf{S}^{\mathbf{0}}_{\ell} = \varpi_{,r}[2\mathbf{W}^{\mathbf{0}}_{\ell} + \mathbf{N}^{\mathbf{0}}_{\ell} + 5\mathbf{L}^{\mathbf{0}}_{\ell} + 2n\mathbf{V}^{\mathbf{0}}_{\ell}P_{\ell,\mu} + 2\mu\mathbf{V}^{\mathbf{0}}_{\ell}P_{\ell,\mu,\mu}] + 2\boldsymbol{\varpi}\mathbf{W}^{\mathbf{0}}_{\ell}(Q-1)\nu_{,r}P_{\ell,\mu},$$

functions describing the polar perturbations at zero order on the angular velocity

function responsible for the dragging of inertial frames

rotating stars extert a dragging not only of the bodies, but also of the waves, and consequently an incoming polar gravitational wave can convert through the fluid oscillations it excites, some of its energy into outgoing axial waves.

Quasi-normal modes frequencies were studied using two different approaches

frequency domain

Thorne and collaborators,
Lindblom, Detweiler,
Chandrasekhar, Ferrari

first calculations for the polar modes in 1983 by Lindblom and Detweiler *Ap.J Suppl **53**,1983*

time domain

Vishveshwara (for black holes),
Andersson, Kokkotas, Stergioulas
Allen, Schutz

Gravitational wave asteroseismology:

Suppose that a gravitational signal emitted by a perturbed neutron star is detected and, by an appropriate data analysis, we are able to determine the frequency of one or more mode: will this information allow to constraints the equation of state of matter in the stellar core?

To answer these question we need to compute the mode frequencies (and the damping times) for stars modeled with the EOS proposed to describe matter at supranuclear density

Different families of modes can be directly associated with different core physics.

f (fundamental)-mode (which should be the most efficient GW emitter) scales with average density

p (pressure)-modes overtones probe the sound speed throughout the star

g (gravity)-modes are sensitive to thermal/composition gradients: the restoring force is buoyancy

w-modes represent oscillations of spacetime (high frequency very rapid damping). They exist both for polar and axial perturbations

s (trapped)-modes (axial modes ; they exist only for utradense stars)

r -modes (Coriols force as restoring force; the star rotates)

A mature neutron star also has elastic shear modes in the crust and superfluid modes.
Magnetic stars may have complex dynamics due to the internal magnetic field.

There is a lot of physics to explore.

The equation of state (EOS) in the interior of a neutron star is largely unknown

At densities lager than $\rho_0 = 2.67 \times 10^{14}\ \mathrm{g/cm^3}$ the fluid is a gas of interacting nucleons

Available EOS have been obtained within models of strongly interacting matter, based on the theoretical knowledge of the underlying dynamics and constrained, as much as possible, by empirical data.

Two main, different approaches:

- nonrelativistic nuclear many-body theory **NMBT**

- relativistic mean field theory **RMFT**

A useful way of classifying EOS's is through their stiffness, which can be quantified in terms of the speed of sound v_s : stiffer EOS's correspond to higher v_s .
stiffer EOS's correspond to less compressible matter.

Non Relativistic Nuclear Many-Body Theory **NMBT**

nuclear matter is viewed as a collection of pointlike protons and neutrons, whose dynamics is described by the nonrelativistic Hamiltonian:

$$H = \sum_i \frac{p_i^2}{2m} + \sum_{j>i} v_{ij} + \sum_{k>j>i} V_{ijk}$$

-The two- and three-nucleon interaction potentials are obtained from fits of existing scattering data.
- ground state energy is calculated using either variational techniques or G-matrix perturbation theory

Relativistic mean field theory **RMFT**

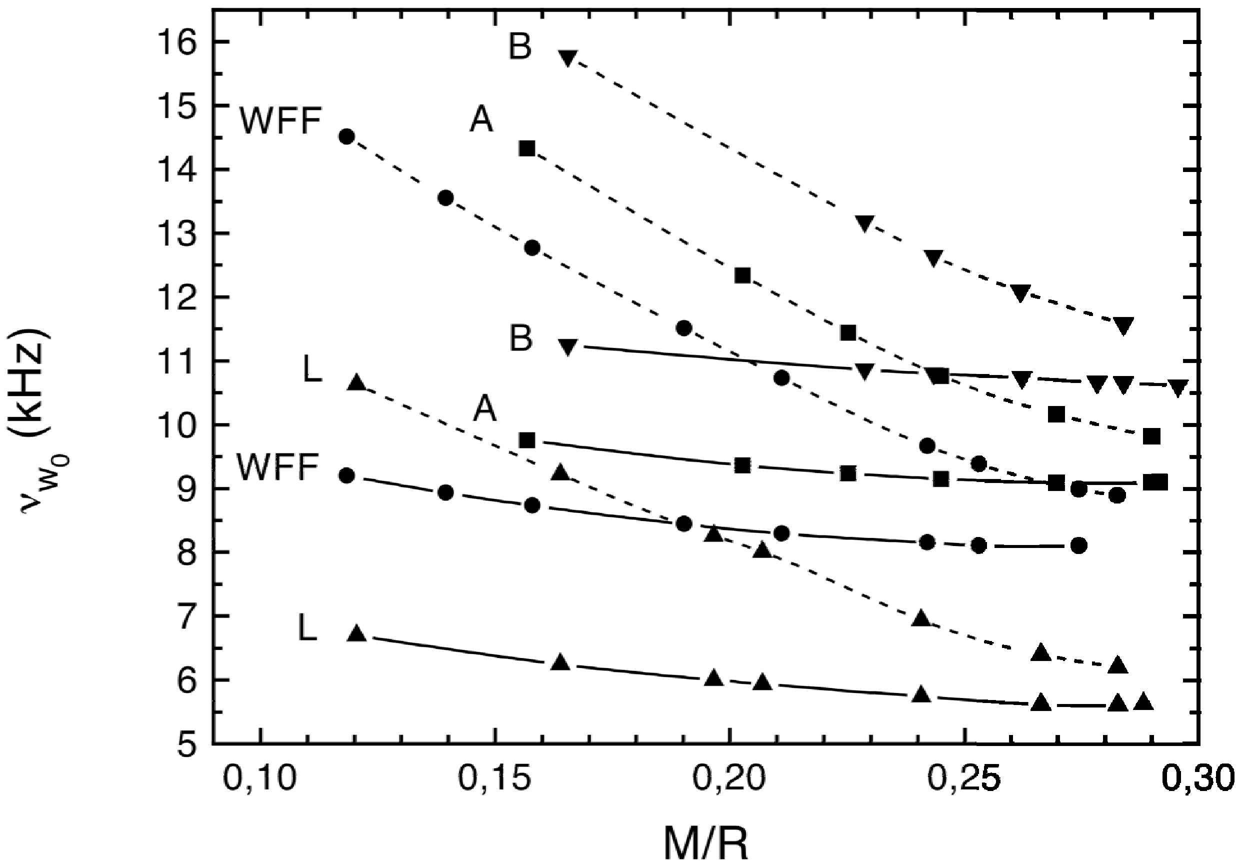
- based on the formalism of relativistic quantum field theory, nucleons are described as Dirac particles interacting through meson exchange. In the simplest implementation of this approach the dynamics is modeled in terms of a scalar and a vector field.
- equations of motion are solved in the mean field approximation, i.e. replacing the meson fields with their vacuum expectation values
- the parameters of the Lagrangian density, i.e. the meson masses and coupling constants, can be determined by fitting the empirical properties of nuclear matter, i.e. binding energy, equilibrium density and compressibility

NMBT and RMFT can be both generalized to account for the appearance of hyperons

Pure spacetime modes

w-modes have high frequency and very rapid damping.

They exist both for polar and axial perturbations



Andersson N., Kokkotas K.D, 1998 MNRAS ***299***

Benhar O., Berti E., Ferrari V. 1999 MNRAS ***310***

dashed lines: polar w-modes

continuous lines: axial w-modes

EOS A . Pandaripande 1971. Pure neutron matter, with dynamics governed by a nonrelativistic Hamiltonian containing a semi-phenomenological interaction potential.

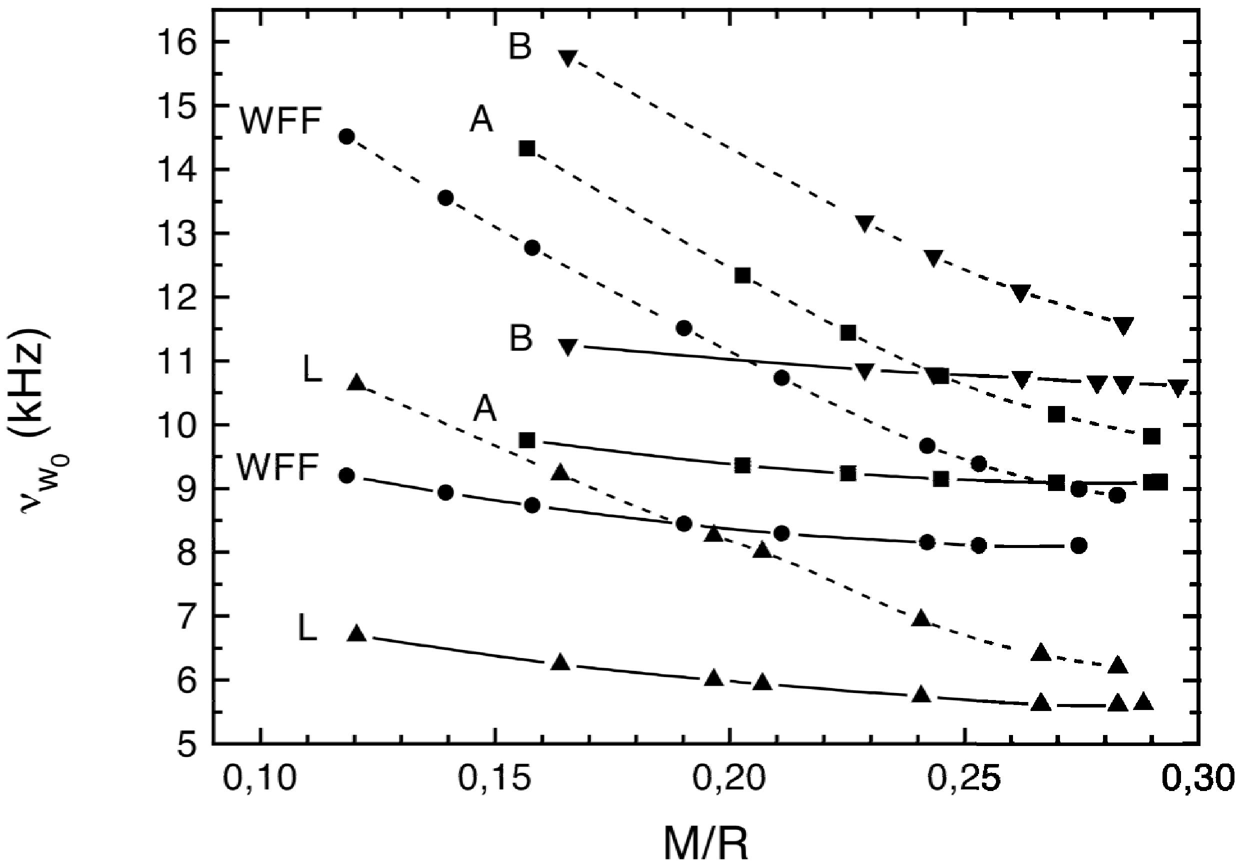
EOS B. 1971 Generalization of EOS A, including protons, electrons and muons in _-equilibrium, as well as heavier barions (hyperons and nucleon resonances) at sufficiently high densities.

EOS WWF. Wiringa, Fiks, Fabrocini 1988. Neutrons, protons, electrons and muons in _-equilibrium. The Hamiltonian includes two- and three-body interaction potentials. The ground state energy is computed using a more sophisticated and accurate many body technique.

EOS L. Pandaripande & Smith1975. Neutrons interact through exchange of mesons (_,_, _). The exchange of heavy particles (_,_) is described in terms of nonrelativistic potentials, the effect of _-meson is described using relativistic field theory and the mean-field approximation.

Pure spacetime modes

w-modes have high frequency and very rapid damping.
They exist both for polar and axial perturbations



about the EOS compressibility:
- B softest (more compressible)
- L stiffest

-axial and polar w-modes depends essentially on the stiffness of the equation of state.

-axial w-mode frequencies range within intervals that are separated;
for each EOS ν_w
is nearly independent of M/R

Andersson N., Kokkotas K.D, 1998 MNRAS ***299***
Benhar O., Berti E., Ferrari V. 1999 MNRAS ***310***

dashed lines: polar w-modes
continuous lines: axial w-modes

unfortunately w-modes frequencies are rather far from the sensitivity area of modern GW detectors

Polar Quasi Normal Modes which are coupled to fluid motion: frequencies are smaller than for those of the w-modes.

For a cold, old neutron star

first calculations in 1983 by Lindblom and Detweiler

Lindblom, Detweiler Ap.J Suppl ***53****,1983*
N. Andersson, K.D. Kokkotas, MNRAS ***299****, 1998*
Benhar, Ferrari ,Gualtieri Phys. Rev. D ***70****, 2004*
Benhar, Ferrari ,Gualtieri, Marassi Gen. Rel. Grav ***39*** *, 2007*

results for the fundamental mode, which is expected to be the most efficient GW emitter:

The fundamental mode frequency of old, cold neutron stars is plotted for different EOS versus the mass of the star.

strange stars (yellow region)
modeled using MIT bag model

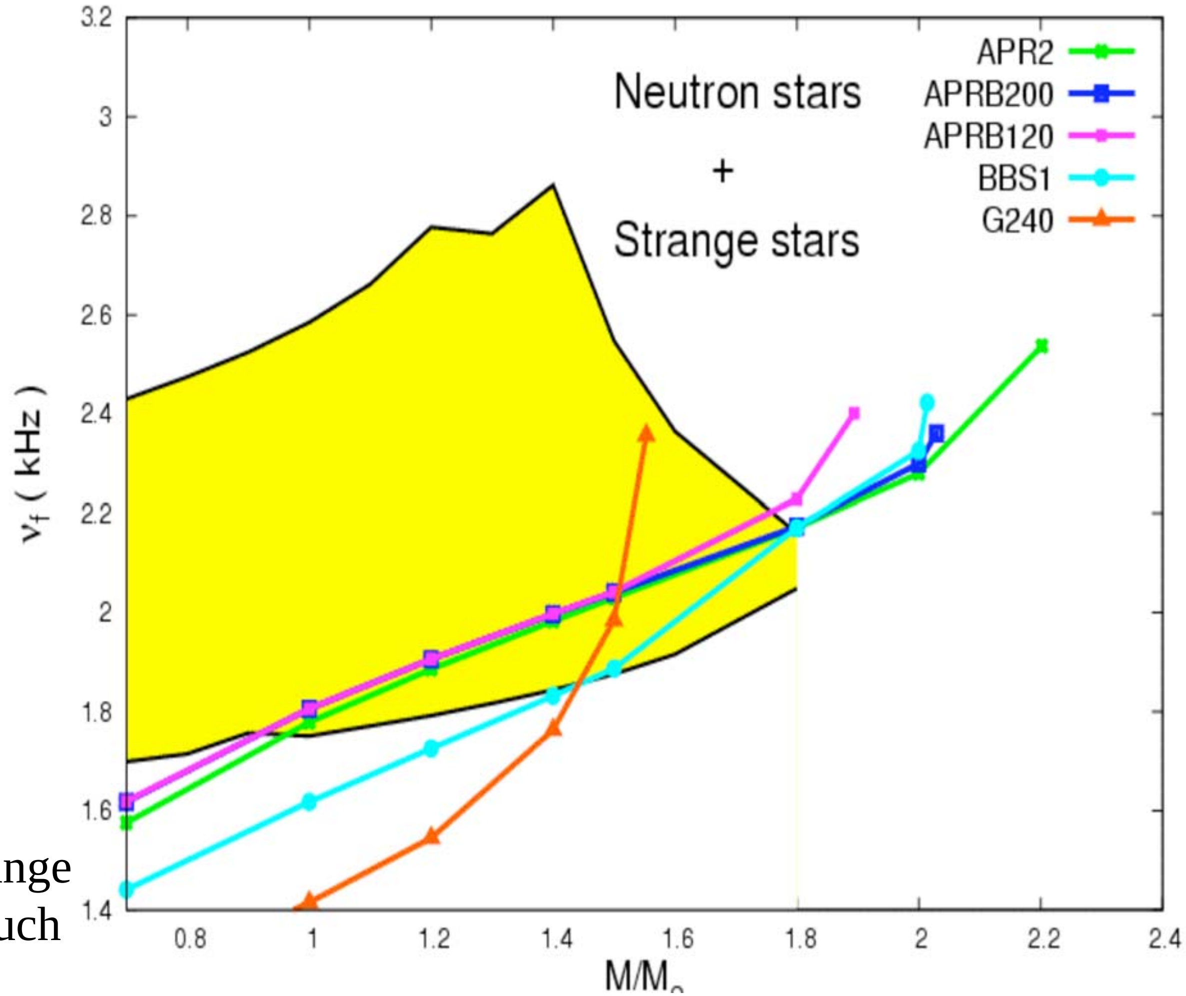
$m_s \in (80 - 155)$ MeV

$\alpha_s \in (0.4 - 0.6)$

$B \in (57 - 95)\ \mathrm{MeV/fm^3}$

*Benhar, Ferrari ,Gualtieri Phys. Rev. D **70**, 2004*
*Benhar, Ferrari ,Gualtieri, Marassi Gen. Rel. Grav **39** , 2007*

- Strange stars cannot emit GWs with _f < 1.7 kHz , for any values of the mass in the range we consider
- There is a small range of frequency where neutron/hybrid stars are indistinguishable from strange stars. However, there is a large frequency region where only strange stars can emit.
- Since _f is an increasing function of the bag constant B, detecting a GW from a strange star would allow to set constraints on B much more stringent than those provided by the available experimental data



Equations of state considered

Inner core $\rho > \rho_0$ ρ_0=2.67 x 10^{14} g/cm^3

APR2:: n p e^- μ^- 3-body interaction phenomenological Hamiltonian

2-body potential= Argonne v18, 3-body potential=Urbana IX

Schroedinger equation solved using variational methods

including relativistic corrections

Akmal A., Pandharipande V.R, Ravenhall D.G., Phys. Rev C58, 1998

APRB120/200: **APR2**+ interacting quarks confined to a finite region (the bag)

whose volume is limited by a pressure **B** said the bag constant

(**B**=120 or 200 MeV/fm^{3,} α_s=0.5 m_s=150 MeV)

BBS1: n p e^- μ^- 3-body interaction phenomenological Hamiltonian

2-body potential= Argonne v18, 3-body potential= Urbana VII

(no relativistic corrections); Schroedinger equation solved using perturbative

methods *Baldo M., Bombaci I., Burgio G.F.*, A&A 328*, 1997*

BBS2: same as BBS1+ **heavy barions** Σ^{--} and $\Lambda^{\mathbf{0}}$ (no relativistic corrections)

Baldo M., Burgio G.F., Schulze H,J, *Phys. Rev. C61, 2000*

G240: e^- μ^- and the complete octet of baryons; mean field approximation is used to

to derive the equations of the fields *Glendenning N.K.*

Can stars be entirely made of quarks **(Bodmer 1971,Witten 1984)**

MIT Bag model :
Fermi gas of up, down, strange quarks confined in a region with volume determined by pressure= **Bag constant B**. The interactions between quarks are treated perturbatively at first order in the coupling constant α_s

From Particle Data Book

$$m_u \sim m_d \sim \text{few MeV} \qquad m_s = (80\text{-}155)\ \text{MeV}$$

3 parameters α_s , m_s , B

- **Hadron collision experiments** $0.4 \leq \alpha_s \leq 0.6$

- **High energy experiments** $57 \leq B \leq 350$ MeV/fm3
  (hadron mass, magnetic moments, charge radii measurements)

- **The requirement that SQM is stable at zero temperature implies that** $B \leq 95$ MeV/fm3

We choose $57 \leq B \leq 95\ \text{MeV/fm}^3$

do we have a chance to detect a signal from an oscillating star?

A typical GW signal from a neutron star pulsation mode has the form of a damped sinusoid

$$h(t) = \boxed{\mathcal{A}}\, e^{-(t-t_0)/\boxed{\tau_d}} \sin[2\pi \boxed{f} (t-t_0)] \quad \text{for } t > t_0$$

$$\boxed{\mathcal{A}} \approx 7.6 \times 10^{-24} \sqrt{\frac{\Delta E_\odot}{10^{-12}} \frac{1\ \mathrm{s}}{\tau_d}} \left(\frac{1\ \mathrm{kpc}}{d}\right) \left(\frac{1\ \mathrm{kHz}}{f}\right) .$$

$$\boxed{\Delta E_\odot = \Delta E_{GW}/M_\odot c^2} = \text{total energy}$$

How much energy would need to be channeled into a mode?

For mature neutron stars we can take as a bench-mark the energy involved in a typical pulsar glitch, in which case

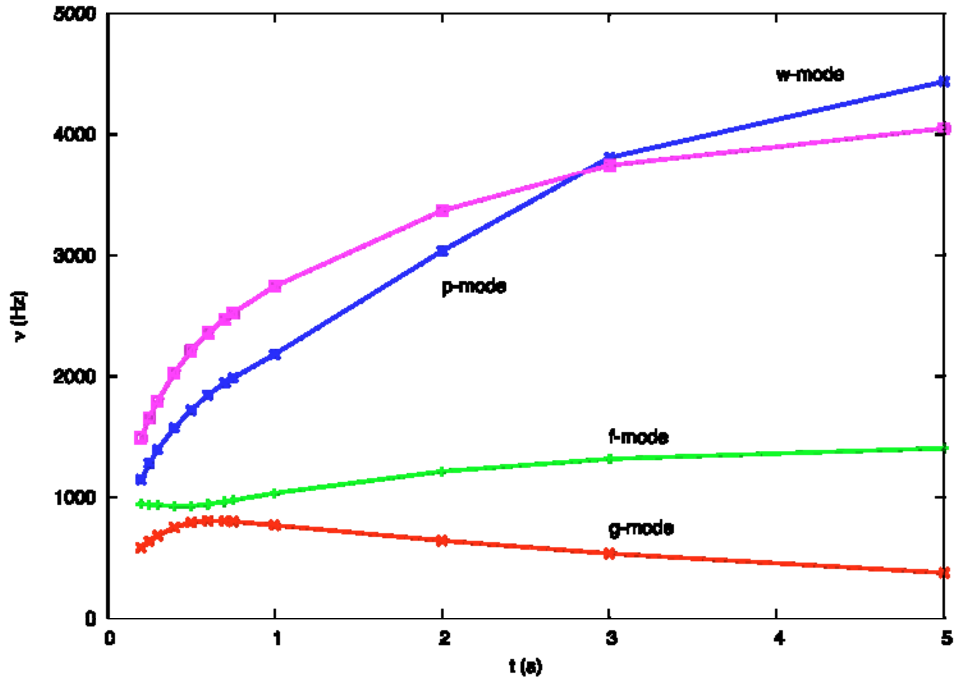
$$\Delta E_{GW} = 10^{-13} M_\odot c^2$$

Assuming $\boxed{f} \sim 1500$ Hz, $\boxed{\tau_d} \sim 0.1$ s, $d = 1$ kpc, $\boxed{\mathcal{A}} \approx 5 \times 10^{-24}$

3rd generation detectors are needed to detect signals from old neutron stars

More promising are oscillations from newly-born neutron stars: more energy can be stored in the modes

The oscillation spectrum evolves during the observation:



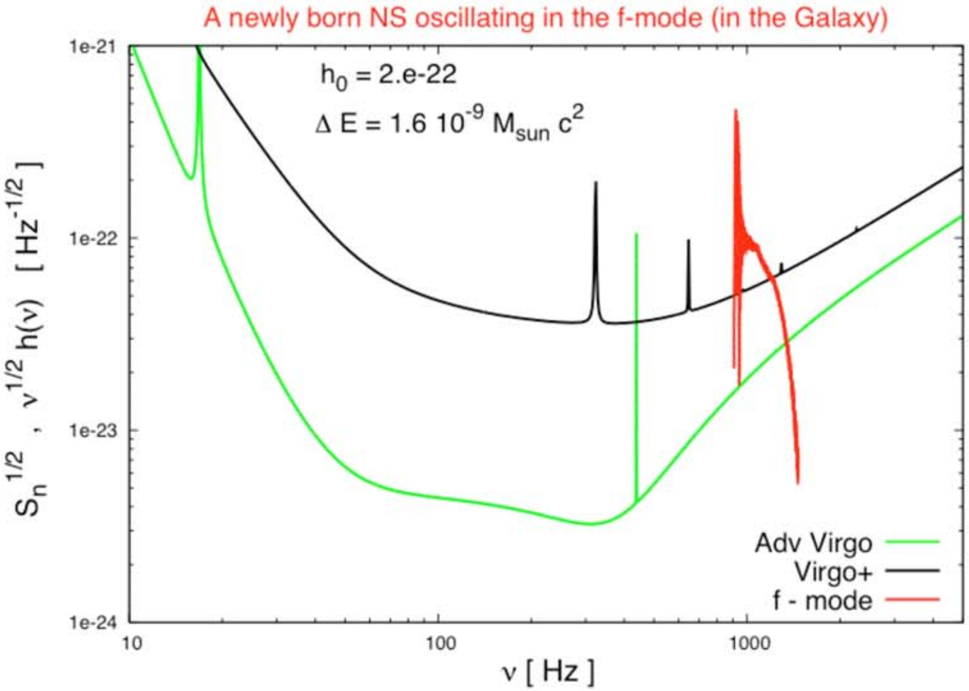
Having _(t) and _(t), we can estimate the amount of energy _E_{GW} that should be stored in a given mode for the signal to be detectable with an assigned SNR by a given detector.

The frequencies of the fundamental mode, and of the first g-, p- and w- modes of an evolving proto--neutron star are plotted as functions of the time elapsed from the gravitational collapse, during the first 5 seconds.

Pons, Reddy, Prakash, Lattimer, Miralles, ApJ ***307*** *1999*

Ferrari ,Miniutti, Pons MNRAS, ***342*** *2004*

If the newly born star is oscillating in the f-mode:



If we assume that

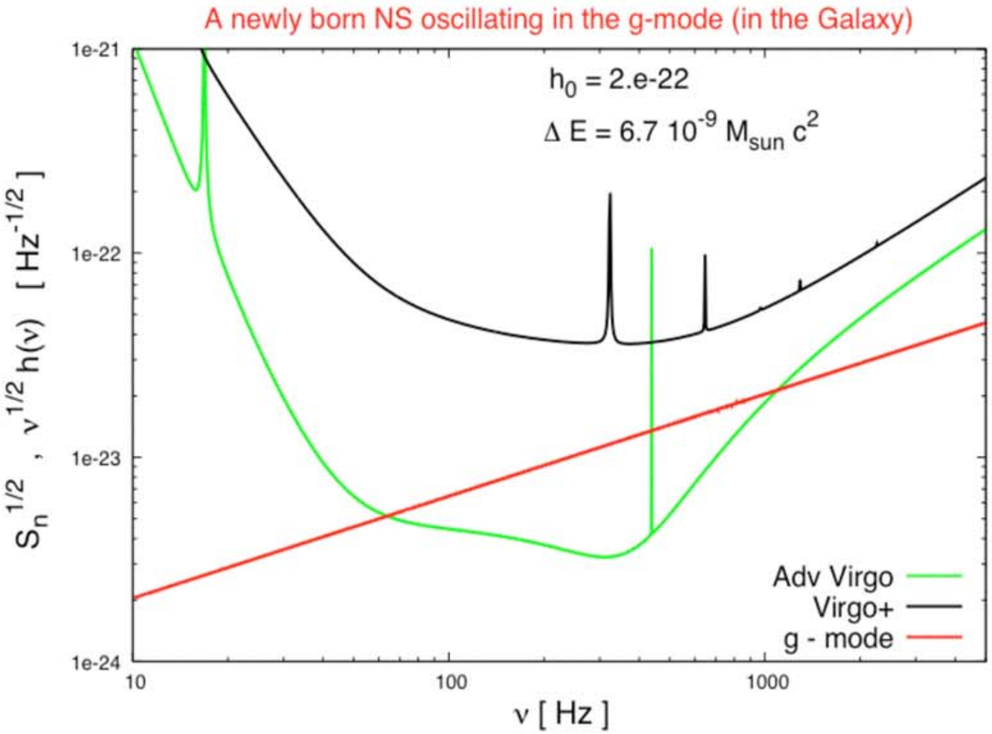
$$\Delta E_{GW} = 1.6 \cdot 10^{-9}\ M_{\odot} c^2$$

is stored into the f-mode

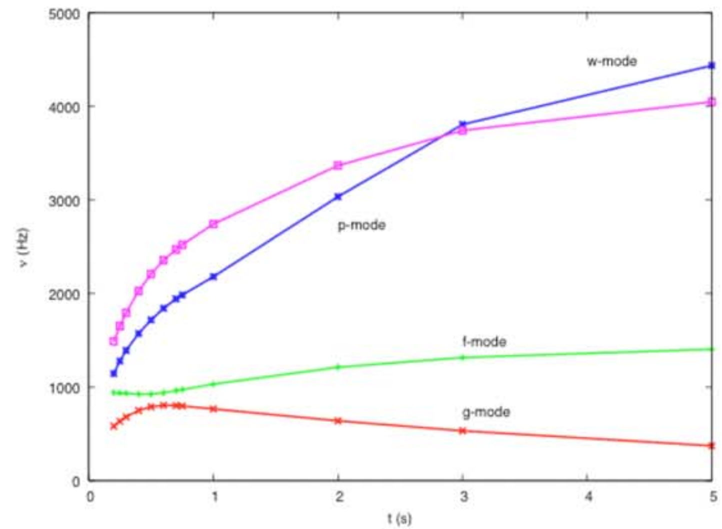
$SNR = 2.7$ by Virgo+

$SNR = 8$ by Advanced Virgo.

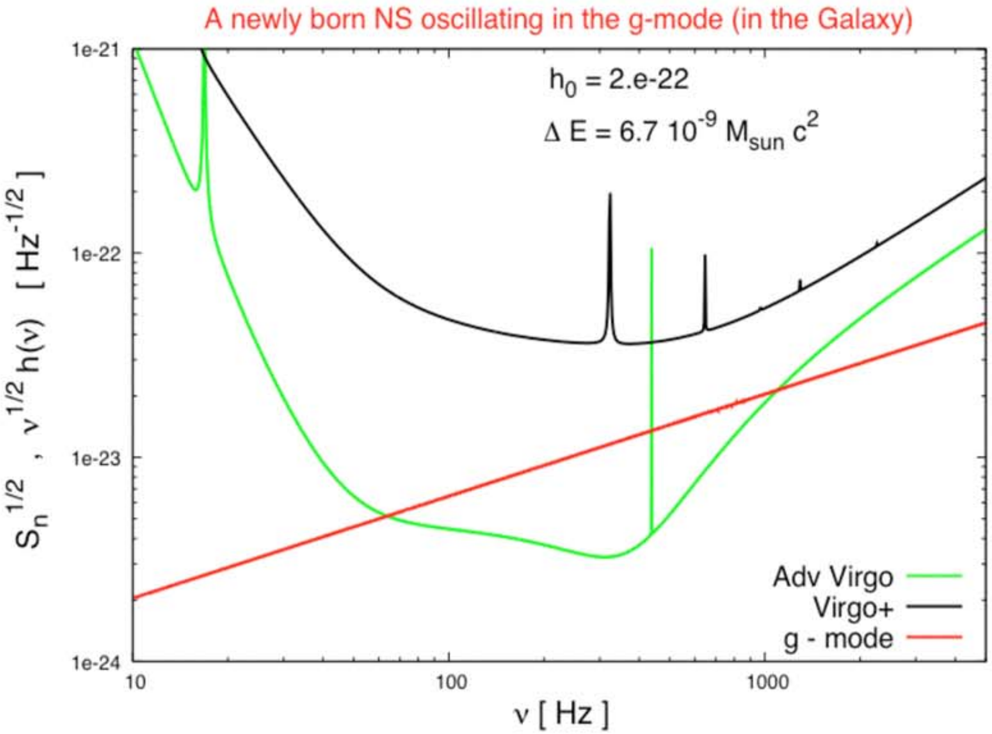
If the newly born star is oscillating in the g-mode:



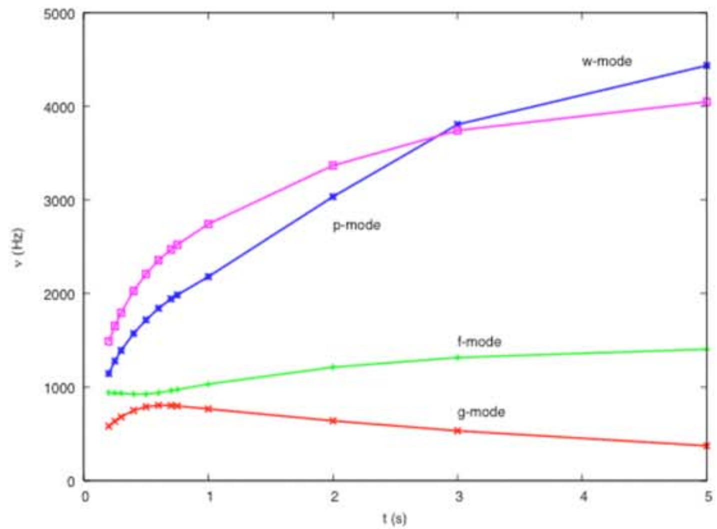
modes evolution



If the newly born star is oscillating in the g-mode:



modes evolution



If we assume that $\Delta E_{GW} = 6.7 \cdot 10^{-9}\ M_{\odot} c^2$ is stored into the g-mode

$SNR = 8$ by Advanced Virgo.

Recent applications of the theory of stellar perturbatons in connection with magnetar observations (B > 10^{14} Gauss)

Soft Gamma Repeaters (SGRs) are astronomical objects with a giant flare activity:
L $\simeq$ 10^{44} -10^{46} ergs/s

giant flares are thought to be associated with starquakes, due to a rearrangements of the the huge magnetic field.

3 events detected, in 1978, 1998, 2004: in the tail of two of them, lasting several hunded seconds, Quasi-Periodic Oscillations have been observed

_ = 18, 26, 30, 92, 150, 625, 1840 Hz in SGR 1860-20
_ = 28, 53, 84, 155 Hz in SGR 1900-14

are these interface modes generated at the interface between core and crust?
they range within 10-30 Hz
are these shear modes in the solid crust of neutron stars?
they start from around 100 Hz
are these Alfven oscillations?

magnetic fields have to be included in the picture

Cowling approximation and time evolution are the most used approaches

The QPO spectrum is not yet fully understood!

groups working on these topics led by Andersson, Kokkotas, Stergioulas, Jones, Watts…

A theory of perturbations for rotating stars

for perturbed black holes, variable separation in terms of:

Schwarzschild black holes: tensorial spherical harmonics	Kerr black holes: oblate spheroidal harmonics

As we have seen in the case of slow rotation, the standard expansion in tensor spherical harmonics leads to a coupling of the axial and polar perturbations:
the number of couplings to be considered increases with the rotation speed

The mathematical tools used so far do not allow to separate the perturbed equations

stellar perturbations of rotating stars have been studied either in the slow rotation regime, or using the Cowling approximation, which neglect the spacetime perturbations.

or, more recently, in full GR!

B. Zink, O. Korobkin, E. Schnetter, N. Stergioulas,   Phys. Rev. D81, 2010

Supercomputers allow us to approach very complex problems, which only ten years ago one would had dreamed to solve.

However

perturbation theory remains a very powerful tool to investigate the dynamical behaviour of stars.

If the harmonics appropriate to separate the equations for the perturbations of a rotating stars are found, the field will receive an incredibly powerful burst.

Polar Quasi Normal Modes: can a NS radius be measured using GWs?

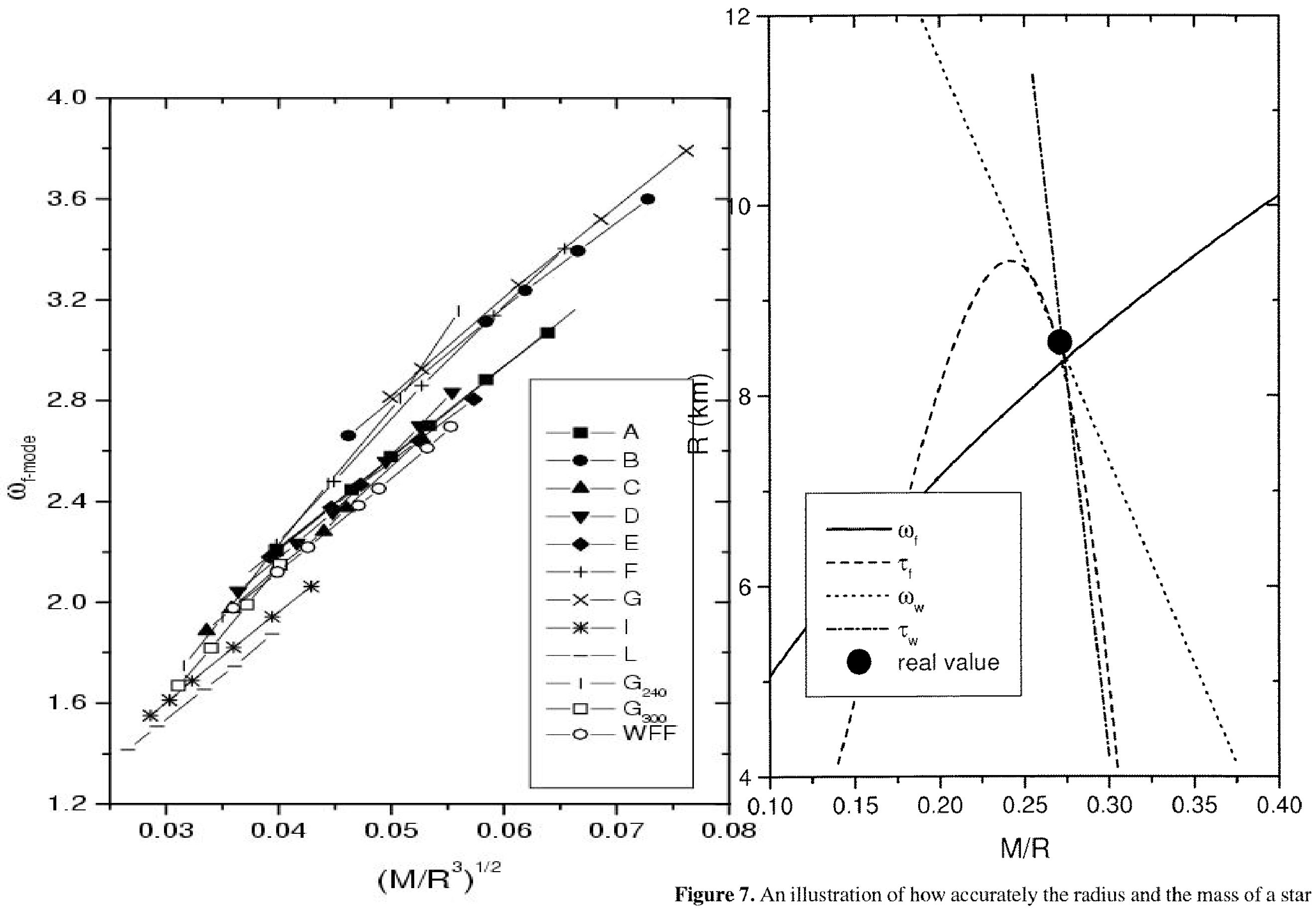


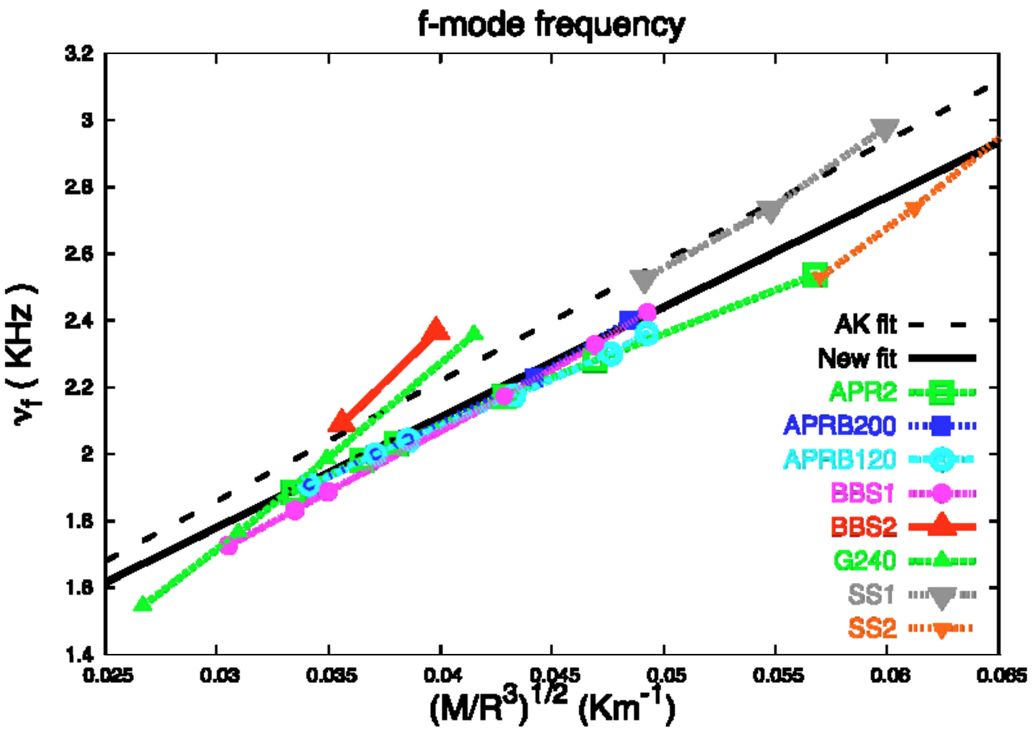
Figure 7. An illustration of how accurately the radius and the mass of a star can be inferred from detected mode data and our empirical relations.

'The results indicate that, should the various pulsation modes be detected by the new generation of gravitational wave detectors that come online in a few years, the mass and the radius of neutron stars can be deduced with errors no larger than a few per cent. '

N. Andersson, K.D. Kokkotas, MNRAS ***299****, 1059, 1998*

$$\omega_f(kHz) \sim 0.78 + 1.653 \left[\left(\frac{M}{1..4\ M_\odot} \right) \left(\frac{10\ km}{R} \right)^3 \right]$$

Polar Quasi Normal Modes: some more recent calculations using new EOSs



$$\nu_f = a + b\sqrt{\frac{M}{R^3}}$$

$$a = 0.79 \pm 0.09, \quad b = 33 \pm 2$$

the new fit (continuous black line) is sistematically lower than the Andersson-Kokkotas fit (dashed line) by about 100 Hz;

this basically shows that the new EOS are, on average, less compressible (i.e. stiffer) than the old ones.

Benhar, Ferrari ,Gualtieri Phys. Rev. D70 n.12, 2004

How to use the fits: consider a NS with

M=1.4 Mo R=11.58 km (EOS APR2)

The frequencies of the fundamental mode and of the first p-mode are

—$_F$ =1.983 kHz, —$_{p1}$ =6,164 kHz

$$\nu_f = a + b\sqrt{\frac{M}{R^3}},$$

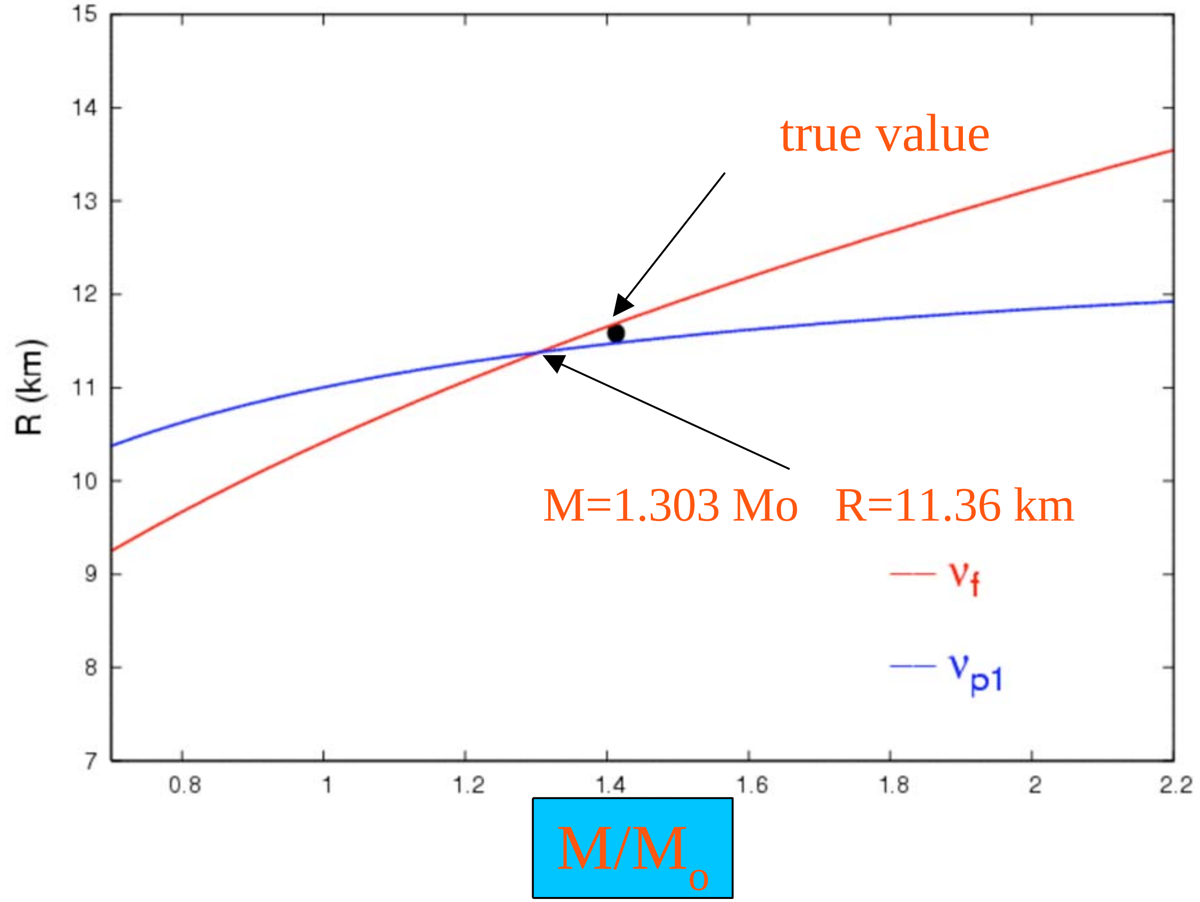
$$a = 0.79 \pm 0.09, \quad b = 33 \pm 2,$$

a in kHz, b in km $\cdot$ kHz.

$$\nu_{p_1} = \frac{1}{M}\left[a + b\frac{M}{R}\right],$$

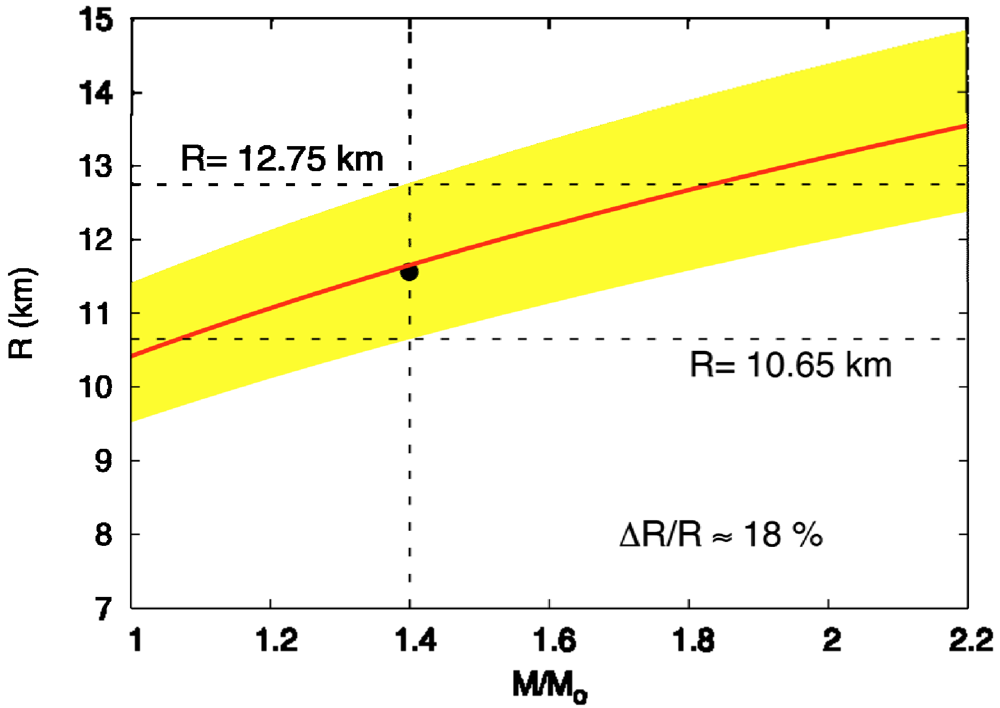
$$a = -1.5 \pm 0.8, \quad b = 79 \pm 4,$$

a and b in km·kHz,



Mass with a 7% error
radius with a 2% error

the new fits come with an error bar:



Uncertainty on radius and mass is huge!
It ranges from 10% to 20% for R

mass cannot be determined

gravitational wave asteroseismology will become possible when GW-detectors will become more sensitive to the high frequency region, and when nuclear matter studies will put tighter constraints on the parameters that characterize the equation of state of superdense matter.